TMUA

COMMUNITY PAPERS

Unattributed TMUA Mock Set A Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Unattributed

Question 1

How many real solutions are there to the equation

$$x|x| + 1 = 3|x|\,?$$

A 0

B 1

C 2

D 3

E 4

Question 2

There is a statement p: "If $x \geq 1$ then $2^x + 1 \geq 3$".

What is the **contrapositive** of the statement p?

A If $2^x + 1 \geq 3$, then $x \geq 1$.

B If $2^x + 1 < 3$, then $x < 1$.

C If $x \geq 1$, then $2^x + 1 < 3$.

D If $x < 1$, then $2^x + 1 \geq 3$.

Question 3

If

$$\log_2(\log_3 x) = \log_3(\log_4 y) = \log_4(\log_2 z) = 0$$

then what is the value of $(x + y + z)$?

A 9

B 8

C 7

D 6

E 5

Question 4

Consider the following statement about the integer n:

If n is even, then n is not a prime.

Which of the following is a **counterexample** to this statement?

I. $n = 0$

II. $n = 2$

III. $n = 3$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 5

Write the expansion of $\left(x^3+x^2+2\right)^4 x$ and find the coefficient of x^4.

A 32

B 4

C 8

D 16

E 30

Question 6

Consider the following claim:

The sum of two rational numbers is a rational number.

Here is an attempted proof of this claim:

I. According to the definition of rational numbers, $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers.

II. Expanding the form $\frac{a}{b} + \frac{c}{d}$, using the rational number definition, we get $\frac{ad + bc}{bd}$.

III. Since both $(ad + bc)$ and bd are integers, and bd is non-zero, according to the definition of rational numbers, $\frac{ad + bc}{bd}$ is a rational number.

Which of the following best describes this proof?

A The proof is completely correct, and the claim is true.

B The proof is completely correct, but there are counterexamples to the claim.

C The proof is wrong, and the first error occurs on line I.

D The proof is wrong, and the first error occurs on line II.

E The proof is wrong, and the first error occurs on line III.

Question 7

Consider the following statements:

P: $x = 1$

Q: $x^2 + x - 6 < 0$

Which of the following is true?

A P is **necessary and sufficient** for Q

B P is **necessary** but **not sufficient** for Q

C P is **sufficient** but **not necessary** for Q

D P is **not necessary** and **not sufficient** for Q

Question 8

The maximum coefficient of the expansion of $\left(\frac{1}{x} + x^2\right)^6$ is a.

How big is the area between the line $y = \frac{3}{20}ax$ and the curve $y = x^2$?

A $\frac{3}{2}$

B $\frac{5}{2}$

C $\frac{7}{2}$

D $\frac{9}{2}$

Question 9

The function $\mathrm{f}(x) = x^3 + ax^2 + bx + c$ has two distinct stationary points at x_1 and x_2, and $\mathrm{f}(x_1) = x_1$.

How many distinct solutions does the equation

$$3\left(\mathrm{f}(x)\right)^2 + 2a\,\mathrm{f}(x) + b = 0$$

have?

A 2

B 3

C 4

D 5

E 6

Question 10

Consider the following statement:

If x is a real number, such that $x^2 \leq x^4$.

Which is the **sufficient** but **not necessary** condition for the statement to be true?

A $x = 1$

B $x < 1$

C $x > -1$

D $x \leq 1$

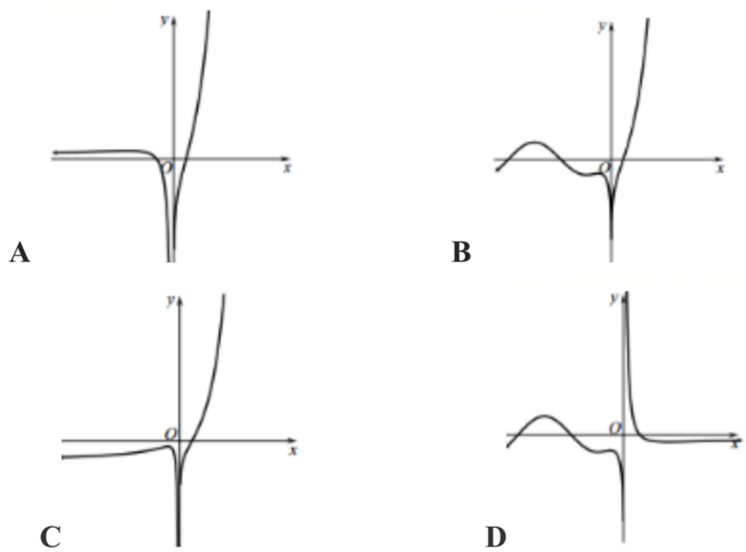
E $x \geq -1$

F $x \geq 1$ and $x \leq -1$

Question 11

The graph of the following function can be:

$$y = \sin x + e^x \ln |x|$$



A A

B B

C C

D D

Question 12

For a positive number a, let

$$I(a) = \int_0^a \left(4 - 2^{x^2}\right) \mathrm{d}x$$

Then $\dfrac{\mathrm{d}I}{\mathrm{d}a} = 0$ when a equals

A $\dfrac{1+\sqrt{5}}{2}$

B $\sqrt{2}$

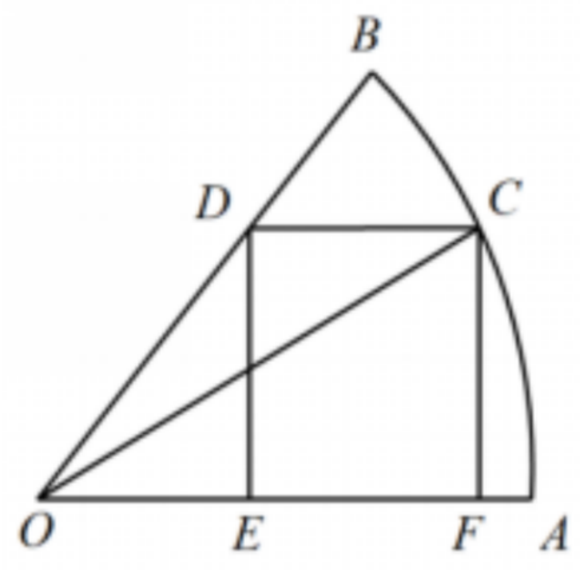
C $\dfrac{1-\sqrt{5}}{2}$

D 1

E 2

Question 13

Given that the radius of the sector AOB is 1 with the centre angle $\frac{\pi}{3}$, and C is a point on the arc as the graph shows, the maximum area of the rectangle is:



A $\frac{\sqrt{3}}{12}$

B $\frac{\sqrt{3}}{6}$

C $\frac{1}{2}$

D 1

E $\sqrt{3}$

Question 14

In a Cartesian coordinate system, define the ”Manhattan distance” between two points as the sum of the absolute differences of their coordinates, so that

$$d(P,\, Q) = |x_1 - x_2| + |y_1 - y_2|$$

is the Manhattan distance between the two points P and Q.

Consider the following statements:

I. The set of points with a Manhattan distance of 1 from the origin forms a circle.

II. The set of points with a Manhattan distance of 1 from the origin forms a square.

III. The set of points whose absolute difference of Manhattan distances from $M(-1,\, 0)$ and $N(1,\, 0)$ is 1 forms two parallel lines.

IV. The set of points whose sum of Manhattan distances from $M(-1,\, 0)$ and $N(1,\, 0)$ is 6 forms a hexagon with an area of 16.

How many statements **must** be true?

A none of them

B 1

C 2

D 3

E 4

Question 15

P is a point moving along the line l: $3x + 4y - 7 = 0$. There are two lines PM and PN from the point P; PM and PN each meet the circle C: $(x+1)^2 + y^2 = r^2$ at exactly one point.

If the maximum value of the angle $\angle MPN$ is $\frac{\pi}{3}$, what is the value of r?

A 4

B 3

C 2

D 1

E none of the above

Question 16

The real numbers m and c are such that the equation

$$x^2 + (mx + c)^2 = 1$$

has a repeated root x, and also the equation

$$(x - a)^2 + (mx + c - 1)^2 = 1$$

has a repeated root x (which is not necessarily the same value of x as the root of the first equation). a is a real number.

Which one of the following is a **sufficient** condition for there to be only 3 possibilities for the line $y = mx + c$?

A $a = \sqrt{3}$

B $a > \sqrt{3}$

C $a \geq \sqrt{3}$

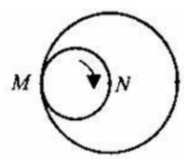
D $a < \sqrt{3}$

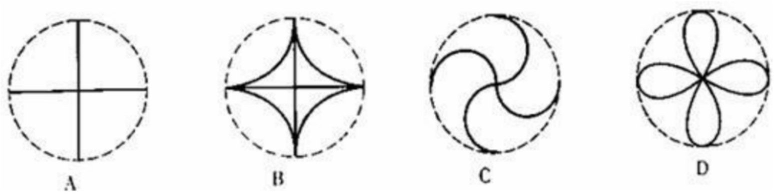
E $a \leq \sqrt{3}$

Question 17

A small circle of diameter 1 rolls anticlockwise along the inner wall of a large circle of diameter 2, where M and N are the two ends of a fixed diameter of the small circle.

When the small circle rolls around the inner wall of the big circle, what would the paths of M and N look like in the big circle?





A none of them

B A

C B

D C

E D

Question 18

There are two real numbers a and b with $a > 0$, $b > 0$. Consider the following statements about the function $\mathrm{f}(x)$.

Which statement(s) is/are true?

I. "$\mathrm{f}(x-a)$ is symmetric about the point $(a, 0)$" is **sufficient and necessary** for "$\mathrm{f}(x)$ is an odd function".

II. "$\mathrm{f}(x-a)$ is symmetric about the line $x = a$" is **sufficient and necessary** for "$\mathrm{f}(x)$ is an even function".

III. "$a = b$" is **sufficient and necessary** for "$\mathrm{f}(x-a)$ and $\mathrm{f}(b-x)$ are symmetric about the y-axis".

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 19

x, a, b are all real numbers. If $(a+2)^2+(b-3)^2=1$, then what is the minimum value of

$$(x-a)^2+(\ln x-b)^2\,?$$

A 3

B 18

C $3\sqrt{2}$

D $3\sqrt{2}-1$

E $19-6\sqrt{2}$

Question 20

Arrange the positive integers into a triangular array as shown in the left diagram. Then, following the given rules, take out the odd numbers in even rows and the even numbers in odd rows to obtain the triangular array shown in the right diagram.

Next, arrange the numbers from the right diagram in ascending order to form a sequence $\{a_n\}$. If $a_n = 2011$, then what is the value of n?

A 1024

B 1025

C 1026

D 1027

E 1028

F 1029