

TMUA

COMMUNITY PAPERS

Zach

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Zach/Cody

Question 1

Let a_n be the area bounded by the curves $x = y$, $x^2 + y^2 = n$ and the positive x-axis. What is $\sum_{n=1}^{100} a_n$?

A $\frac{250\pi}{2}$

B $\frac{50\pi}{4}$

C $\frac{5050\pi}{8}$

D $\frac{3805\pi}{8}$

E $150\sqrt{2}\pi$

Question 2

The symbol $\{x\}$ denotes the fractional part of x . For example $\{10.4\} = 0.4$, $\{2\} = 0$. Given that $f(x) = 5x^3 + 2x^2 + 8x + 1$, find

$$\int_0^n f(\{x\} + 1) \, dx$$

where n is a positive integer.

A $\frac{425n}{13}$

B $\frac{1}{2}n(n+1)$

C $\frac{83n}{12}$

D $\frac{437n}{12}$

E $15n^2 + 4n + 8$

Question 3

Let x and y be real numbers such that $x^2 + y^2 = 49$. What is the greatest value of $12x - 5y$?

- A 91
- B 84
- C 104
- D 89
- E 87

Question 4

For what values of x is

$$x + 3 = 1 + 1 + x + 1 + 2x + x^2 + 1 + 3x + 3x^2 + x^3 + \dots$$

A -1

B 0

C $\frac{-3-\sqrt{5}}{2}$

D $\frac{-3+\sqrt{5}}{2}$

E $\frac{-3-\sqrt{5}}{2}$ and $\frac{-3+\sqrt{5}}{2}$

Question 5

How many four-digit square numbers are multiples of 3, 4 and 5?

- A** 0
- B** 1
- C** 2
- D** 3
- E** 4
- F** 5
- G** 6

Question 6

Two identical ice sculptures are left to melt outside. Sculpture A melts in 6 hours. Sculpture B melts in 12 hours. Assuming each sculpture melts uniformly, after how long will Sculpture B have twice as much ice remaining as Sculpture A?

- A 3 hours
- B 3.5 hours
- C 4 hours
- D 4.5 hours
- E 5 hours

Question 7

Alvin and Simon can create a song in 5 days. Alvin and Theodore can create a song in 8 days. Simon and Theodore can create a song in 10 days. Approximately how many days does it take Alvin, Simon and Theodore to make a song together?

- A 3
- B 3.2
- C 3.5
- D 3.7
- E 4.7

Question 8

Find x such that

$$\ln(x \ln(x \ln(x \ln(x \ln(x \cdots = e$$

A e

B e^3

C e^{e+1}

D e^e

E e^{e-1}

Question 9

Find the coefficient of x^{15} in the expansion of $(2x^3 + 3x + 4x^5)^5$.

- A 11280
- B 9920
- C 8462
- D 12482
- E 10592

Question 10

Find the units digit of $34657^{267} + 81$.

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H 7

I 8

J 9

Question 11

There are 129 coins on a table. 128 are fair, and one has heads on both sides. A coin is tossed 8 times and lands on heads every time. What is the probability it lands on heads if thrown a 9th time?

A $\frac{1}{2}$

B $\frac{9}{10}$

C $\frac{7}{8}$

D $\frac{5}{6}$

E $\frac{2}{3}$

Question 12

Find the sum of the solutions to

$$16^{\log_2 x} - 10 \times 27^{\log_3 x} + 35 \times 64^{\log_8 x} - 50x + 24 = 0$$

- A** 0
- B** 8
- C** 10
- D** 24
- E** 43

Question 13

How many solutions does

$$\sin x + \sin^2 x + \sin^3 x + \cdots = \tan x$$

have for $0 \leq x \leq 2\pi$?

A 1

B 2

C 3

D 4

E 5

F 6

Question 14

How many sets of positive integers (a, b, c) with $a \leq b \leq c$ satisfy

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1?$$

- A** 1
- B** 2
- C** 3
- D** 4
- E** 0
- F** ∞

Question 15

Given

$$f(x) = \frac{x\sqrt{x} - 2\sqrt{x}}{\sqrt{2} - \sqrt{x}},$$

find $f'(2)$.

- A** 0
- B** $\frac{1}{\sqrt{2}}$
- C** $\frac{1}{2}$
- D** $-\frac{3}{2}$
- E** $\frac{3}{2}$

Question 16

A sheep called Shaun lives in a circular pen of radius 8. His position inside the pen is random. What is the probability that Shaun is within 1 m of a side of the pen?

A $\frac{1}{2}$

B $\frac{49}{64}$

C $\frac{7}{8}$

D $\frac{15}{64}$

E $\frac{1}{8}$

Question 17

Solve:

$$\log_{x-2} 9 = \log_x 81$$

- A** $x = 1$
- B** $x = 4$
- C** $x = 1$ and $x = 4$
- D** No real solutions
- E** All of the above

Question 18

For which function is

$$f(f(f(f(f(x))))) \neq 0?$$

A $f(x) = \sin x$

B $f(x) = x^7$

C $f(x) = e^x - 1$

D $f(x) = \cos^2 x$

Question 19

A proper divisor of a number is a positive factor that is smaller than the number.

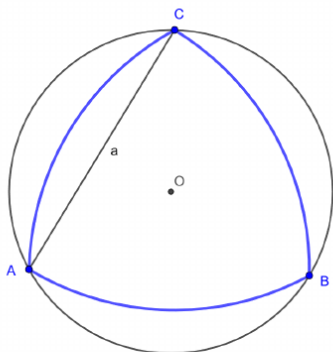
E.g. The proper divisors of 6 are 1,2,3

How many proper divisors does 30^4 have?

- A** 63
- B** 125
- C** 124
- D** 64
- E** None of the above

Question 20

Given that CB is an arc formed from the circle of radius a centred at A, and AB and AC are defined similarly, what is the area of the shape ABC in terms of a ?



- A $\frac{a^2}{2} \left(\pi - 3 \tan \frac{\pi}{6} \right)$
- B $\frac{a^2}{2} \left(\pi - 3 \tan \frac{\pi}{3} \right)$
- C $\frac{a^2}{2} \left(\pi - 3 \sin \frac{\pi}{6} \right)$
- D $\frac{a^2}{2} \left(\pi - 3 \sin \frac{\pi}{3} \right)$
- E $3a^2 \pi \tan \left(\frac{\pi}{6} \right)$