



tmua.co.uk TMUA Mock Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: tmua.co.uk

Question 1

The sum of the first n terms of a sequence is given by the formula $S_n = 5n^2 + kn$, where k is a constant. The 6th term of the sequence is 52. Find the value of k .

A -13

B -8

C -3

D 3

E 22

Question 2

Let $P(x)$ be defined by:

$$P(x) = x^{2027} - 4x^{2026} + 6$$

When $P(x)$ is divided by $x^2 - 1$, the remainder is $R(x) = ax + b$. What is $R(x)$?

- A** $(x + 2)$
- B** $(x - 2)$
- C** $(-x + 2)$
- D** $(2x + 1)$

Question 3

Find the value of the constant k such that the equation

$$(x^2 - 4x)^2 - 2k(x^2 - 4x) + k^2 - 9 = 0$$

has exactly three distinct real solutions for x .

A -7

B -5

C -1

D 3

E 5

F 7

Question 4

A sequence u_n is defined by $u_0 = 3$ and the recurrence relation

$$u_{n+1} = 1 - \frac{1}{u_n}$$

for $n \geq 0$. Find the value of the sum $S = \sum_{n=0}^{25} u_n$.

A $\frac{51}{2}$

B 26

C $\frac{85}{3}$

D $\frac{57}{2}$

E 29

F 37

Question 5

Consider the infinite series

$$S = \frac{9}{19} + \frac{99}{19^2} + \frac{999}{19^3} + \frac{9999}{19^4} + \dots$$

In each term, the numerator is formed by writing the digit 9 repeatedly, and the denominator is a higher power of 19.

The series S converges.

What is the exact value of S ?

A $\frac{9}{19}$

B 1

C $\frac{20}{19}$

D $\frac{19}{18}$

E $\frac{10}{9}$

F $\frac{19}{9}$

Question 6

In how many ways can 729 be written as a sum of two or more consecutive positive integers?

- A 0
- B 5
- C 6
- D 7
- E 8

Question 7

Let $P(x) = (2x^2 - 3x + 2)^3$.

What is the sum of the coefficients of the even powers of x in the expansion of $P(x)$? (Count the constant term as an even power of x .)

A -171

B 1

C 8

D 171

E 172

F 343

G 344

Question 8

For integers n with $0 \leq n \leq 20$, consider the expression $\log_2 \left(\frac{n-9}{n-2} \right)$. For how many such integers n is this expression defined as a real number and has an integer value?

- A** 0
- B** 1
- C** 2
- D** 3
- E** 4

Question 9

The curve C has equation $y = x^3 - 6x^2 + 4x$. The tangent to C at the point where $x = 1$ intersects C again at the point P . The tangent to C at P intersects C again at the point Q . Determine the x -coordinate of Q .

A -8

B -4

C -2

D 2

E 4

F 8

Question 10

What is the shortest distance between the graphs of the following two equations in the xy -plane?

$$|x| + |y| = 6$$

$$|x - 1| + |y - 4| = 13$$

- A 1
- B $\sqrt{2}$
- C 2
- D $\sqrt{10}$
- E $\sqrt{17}$
- F 6
- G $7\sqrt{2}$

Question 11

The sum S_n is defined by

$$S_n = \sum_{r=1}^n \log_2 \left(1 + \frac{2r+3}{r(r+2)} \right)$$

Find the smallest integer n such that $S_n \geq 4 + \log_2 5$.

- A** 7
- B** 8
- C** 12
- D** 13
- E** 14
- F** 15
- G** 19

Question 12

Consider the equation:

$$4 \sin x \cos x + 2\sqrt{2} \cos x = k(2 \sin x + \sqrt{2})$$

Find the sum of the squares of all possible real values for the constant k such that the equation has exactly three distinct solutions for x in the range $0 \leq x < 2\pi$.

- A** 3
- B** 4
- C** 6
- D** 8
- E** 10
- F** 12

Question 13

Let $f(y) = y^2 - 2$.

Find the number of distinct real solutions x to the equation $f(f(f(|x|))) = 0$

A 2

B 4

C 6

D 8

E 12

F 16

Question 14

Let a and b be positive real numbers. For every real x the following inequality holds: $a \leq \frac{4}{1+e^x} + \frac{1}{1+e^{-x}} + 1 \leq b$. What is the minimum possible value of $b - a$?

- A** 1
- B** 2
- C** 3
- D** 4
- E** 5
- F** 21
- G** 23

Question 15

For a real number x , let $[x]$ denote the largest integer less than or equal to x .

Find the value of

$$\int_{-4}^4 (6x^2 + [x^3]) \, dx$$

A -8

B -4

C 124

D 252

E 256

F 260

Question 16

Evaluate the sum

$$\frac{1}{\log_2(64!)} + \frac{1}{\log_3(64!)} + \frac{1}{\log_4(64!)} + \cdots + \frac{1}{\log_{64}(64!)}$$

- A** 1
- B** 2
- C** 4
- D** 8
- E** 16
- F** 32
- G** 64

Question 17

Let

$$S = \sum_{r=0}^{19} (-1)^r \binom{32}{r} \binom{32}{13+r}.$$

Evaluate S .

- A** 0
- B** $\binom{32}{9}$
- C** $\binom{32}{10}$
- D** $\binom{32}{13}$
- E** $\binom{32}{15}$
- F** $\binom{32}{16}$

Question 18

Let θ be a real number. Define two infinite series:

$$S_1(\theta) = \sin^2 \theta + \sin^4 \theta + \sin^6 \theta + \dots$$

$$S_2(\theta) = \cos^2 \theta + \cos^4 \theta + \cos^6 \theta + \dots$$

Both series converge whenever $|\sin \theta| < 1$ and $|\cos \theta| < 1$.

Suppose that

$$S_1(\theta) = \frac{2}{3} \quad \text{and} \quad S_2(\theta) = \frac{3}{2}$$

How many distinct real values of θ are there in the range $-2\pi \leq \theta \leq 2\pi$ which satisfy both equations?

- A** 2
- B** 4
- C** 6
- D** 8
- E** 10

Question 19

Find the sum of all distinct real solutions to the following equation:

$$(x^2 - 7x + 11)^{x^2 - 2x - 2} = 1$$

A 7

B 9

C 11

D 13

E 14

F 16

Question 20

Let the “bimode” of a data set be equal to the regular mode if there is a singular most common value, but equal to the mean of the modes if there is no unique modal value.

What is the sum of all real values of x for which the mean, median and bimode (in any order) of the seven numbers

$$-2, 1, 1, 9, 9, 12, x$$

form an arithmetic sequence?

- A** -46
- B** 5
- C** 10
- D** 15
- E** 20
- F** 25
- G** 61
- H** 66