Test of Mathematics for University Admission

Paper 1

Mock Examination Practice

Instructions for Candidates:

- There are 20 questions in this paper.
- Each question is followed by multiple options.
- Choose the single best option for each question.
- Calculators and dictionaries are NOT permitted.
- Use the blank space on each page for your working out.
- Do not turn over until you are told to do so.

Written by Y&A Tutoring

Cambridge Maths and Economics offer holders, TMUA 8.3 to 9.0 (October 2025)

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Question 1

Consider the points (x, y) that lie on the circle with equation $(x-2)^2 + (y-3)^2 = 4$.

What is the largest possible value that $x + y$ can take?

A) $5 + \sqrt{2}$

B) $5 + 2\sqrt{2}$

C) 7

D) $5 + 4\sqrt{2}$

E) 9

Question 2

The coefficient of x^5 in the binomial expansion of $(kx^2 - 2x + 1)^4$ is -32. Find all possible real values of the constant k.

A) $k = 1$ or $k = -2$

B) $k = \frac{1}{2}$ or $k = -3$

C) $k = \frac{4}{3}$ or $k = -1$

D) $k = 2$ or $k = -\frac{1}{3}$

E) $k = \frac{2}{3}$ or $k = -2$

Question 3

What is the shortest possible distance between the circle $(x-3)^2+(y+2)^2=8$ and the straight line $y=x+3$?

A) $\sqrt{2}$

B) 2

C) $2\sqrt{2}$

D) $3\sqrt{2}$

E) $4\sqrt{2}$

Question 4

A student uses the trapezium rule with 2 strips of equal width to estimate the value of the definite integral:

$\int_0^2 \sqrt{4 - x^2} dx$

By comparing this trapezium rule estimate to the exact value of the integral, and considering the shape of the curve, which of the following inequalities is rigorously proven to be true?

A) $\pi < 1 + \sqrt{3}$

B) $\pi > 1 + \sqrt{3}$

C) $\pi < 2 + 2\sqrt{3}$

D) $\pi > 2 + 2\sqrt{3}$

E) $\pi = 1 + \sqrt{3}$

Question 5

A sequence is defined by $u_n = 2^{3n-1}$ for integers $n \geq 1$. Which of the following statements correctly describes the sequence defined by $v_n = \log_4(u_n)$?

A) It is an arithmetic sequence with a common difference of 3.

B) It is an arithmetic sequence with a common difference of 1.5.

C) It is a geometric sequence with a common ratio of 1.5.

D) It is a geometric sequence with a common ratio of 3.

E) It is neither an arithmetic sequence nor a geometric sequence.

Question 6

How many solutions does the equation $\cos(\pi \sin x) = 0$ have in the interval $0 \leq x \leq 2\pi$?

A) 2

B) 3

C) 4

D) 6

E) 8

Question 7

Let $f(x) = 2x^3 - 9x^2 + 12x$ and $g(x) = c$, where c is a real constant.

Let m be the number of distinct real solutions to the equation $f(x) = g(x)$, and let n be the number of distinct real solutions to the equation $f'(x) = g'(x)$.

What is the maximum possible value of $m + n$?

A) 2

B) 3

C) 4

D) 5

E) 6

Question 8

Let $P_1(x)$, $P_2(x)$ and $P_3(x)$ be three quadratic polynomials with real coefficients. The polynomial $Q(x) = P_1(x)P_2(x)P_3(x)$ has exactly 5 distinct real roots.

Let Δ_i be the discriminant of $P_i(x)$ for $i = 1, 2, 3$. Which of the following statements must be true?

A) $\Delta_1\Delta_2\Delta_3 < 0$

B) $\Delta_1\Delta_2\Delta_3 = 0$

C) $\Delta_1\Delta_2\Delta_3 > 0$

D) $\Delta_1\Delta_2\Delta_3 \geq 0$

E) There is insufficient information to determine a correct answer.

Question 9

A quadratic function $f(x)$ has a minimum value of 4 at $x = 2$. Another quadratic function $g(x)$ has a maximum value of 4 at $x = 2$.

How many distinct real roots does the equation $f(x) - g(x) = 0$ have?

A) 0

B) 1

C) 2

D) 3

E) Infinitely many

Question 10

Three fair six-sided dice are rolled. What is the probability that the three numbers rolled can form the side lengths of a non-degenerate triangle?

A) 35/72

B) 1/2

C) 37/72

D) 5/9

E) 41/72

Question 11

In triangle ABC, the side lengths are $BC = a$, $CA = b$ and $AB = c$. It is given that the side lengths satisfy the algebraic relationship:

$a^2 + b^2 = c^2 + ab$

Which of the following expressions gives the exact area of triangle ABC in terms of a and b?

A) $\frac{1}{4}ab$

B) $\frac{1}{2}ab$

C) $\frac{\sqrt{3}}{4}ab$

D) $\frac{\sqrt{3}}{2}ab$

E) ab

Question 12

Find the number of real pairs (x, y) that satisfy the following simultaneous equations:

$3^x + 9^y = 10$

$3^y + 9^x = 10$

A) 0

B) 1

C) 2

D) 3

E) 4

Question 13

Four straight lines are drawn on a 2D plane. None of the lines are parallel to each other, and no three lines intersect at a single point.

Into how many distinct regions do these four lines divide the plane?

A) 8

B) 10

C) 11

D) 14

E) 16

Question 14

Find the sum of all valid solutions to the following equation in the interval $0 \leq x \leq 2\pi$:

$\sin^3 x = (\sin^2 x - 1)\cos x$

A) π

B) 2π

C) $\frac{5\pi}{2}$

D) 3π

E) $\frac{7\pi}{2}$

Question 15

Let $f(x) = x^3 - 6x^2 + 12x - 5$. What is the exact value of $f(2 - \sqrt[3]{5}) + f(2 + \sqrt[3]{5})$?

A) 0

B) 3

C) 6

D) 8

E) 12

Question 16

A function f is defined for all real numbers and has the property that $f(x+y) = f(x)f(y)$ for all real numbers x and y.

It is given that there exists at least one real number a such that $f(a) \neq 0$.

If $f(4) = 16$ which of the following is the only possible value for $f(-2)$?

A) -4

B) $-\frac{1}{4}$

C) $\frac{1}{16}$

D) $\frac{1}{4}$

E) 4

Question 17

A bag contains exactly 3 red marbles and n blue marbles (where $n \geq 2$). Three marbles are drawn at random without replacement.

Given that the probability of drawing exactly 2 red marbles is equal to the probability of drawing exactly 1 red marble, what is the value of n?

A) 2

B) 3

C) 4

D) 5

E) 6

Question 18

Find the sum of all positive integers n such that the expression:

$\frac{n^2+7n+22}{n+3}$

evaluates to an integer.

A) 7

B) 9

C) 10

D) 12

E) 14

Question 19

Find the complete set of values of x for which the following inequality holds:

$\frac{2x-1}{x+2} > x$

A) $x < -2$

B) $x \leq -2$

C) $x > -2$

D) $x \geq -2$

E) All real x

Question 20

Let $f(x) = \frac{x}{\sqrt{1+x^2}}$

Let $f_n(x)$ denote the n-th iteration of $f(x)$ such that $f_2(x) = f(f(x))$, $f_3(x) = f(f(f(x)))$, and so on.

Evaluate $f_{15}(1)$.

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{15}$

E) $\frac{1}{16}$

Answer Key & Brief Hints

1. B. The circle has centre $(2,3)$ and radius 2, so $x+y=5+2(\cos\theta+\sin\theta)=5+2\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$, whose maximum is $5+2\sqrt{2}$. Equivalently, the line $x+y=k$ meets the circle for $|k-5|\leq 2\sqrt{2}$.

2. E. Two products contribute a term in x^5 when $(kx^2-2x+1)^4$ is expanded. Choosing $(kx^2)^2(-2x)$ gives $12k^2(-2)=-24k^2$, and choosing $(kx^2)(-2x)^3$ gives $4k(-8)=-32k$. Setting the total equal to -32 gives $3k^2+4k-4=0$, with roots $k=\frac{2}{3}$ and $k=-2$.

3. C. The perpendicular distance from the centre $(3,-2)$ to $x-y+3=0$ is $\frac{|3+2+3|}{\sqrt{2}}=4\sqrt{2}$. Subtracting the radius $\sqrt{8}=2\sqrt{2}$ leaves $2\sqrt{2}$.

4. B. The integral is a quarter circle of radius 2, so its exact value is π. With 2 strips, $h=1$ and the ordinates are 2, $\sqrt{3}$, 0, giving $T=\frac{1}{2}\left[2+2\sqrt{3}+0\right]=1+\sqrt{3}$. The curve is concave, so each chord lies strictly below it and the estimate is a strict underestimate. Hence $\pi>1+\sqrt{3}$.

5. B. $v_n=\log_4(2^{3n-1})=\frac{1}{2}(3n-1)=1.5n-0.5$, which is linear in n: arithmetic with common difference 1.5.

6. C. $\cos(\pi\sin x)=0$ requires $\pi\sin x=\frac{\pi}{2}+k\pi$, so $\sin x=\frac{1}{2}+k$. Only $k=0$ and $k=-1$ keep $|\sin x|\leq 1$, giving $\sin x=\pm\frac{1}{2}$ and hence the four solutions $\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$.

7. D. Since $g'(x)=0$ and $f'(x)=6(x-1)(x-2)$, the second equation always has exactly $n=2$ solutions. The turning points of f are $(1,5)$ and $(2,4)$, so any c with $4<c<5$ gives $m=3$. The maximum of $m+n$ is 5.

8. D. No discriminant can be negative: a quadratic factor with no real roots would leave only two quadratics, hence at most 4 distinct roots, not 5. Both remaining cases occur. Two quadratics with distinct roots and one repeated root give $2+2+1=5$ and product 0; three quadratics with 6 roots, exactly two of which coincide, give 5 distinct roots and a positive product. So only $\Delta_1\Delta_2\Delta_3\geq 0$ must hold.

9. B. A minimum value of 4 at $x=2$ gives $f(x)=a(x-2)^2+4$ with $a>0$, and a maximum value of 4 at $x=2$ gives $g(x)=b(x-2)^2+4$ with $b<0$. Then $f(x)-g(x)=(a-b)(x-2)^2$ with $a-b>0$, so the only solution is $x=2$, a repeated root. The curves touch at $(2,4)$ and nowhere else, so there is exactly one distinct real root.

10. C. Count the failures instead. If $a+b\leq c$ with c the largest value then $a,b<c$, so the largest value is strictly unique and can sit on any of the 3 dice. For a fixed c the number of pairs with $a+b\leq c$ is $\frac{1}{2}c(c-1)$, and summing over $c=1,\ldots,6$ gives $0+1+3+6+10+15=35$. So $3\times 35=105$ of the 216 rolls fail, and the probability of a genuine triangle is $1-\frac{105}{216}=\frac{111}{216}=\frac{37}{72}$.

11. C. The cosine rule gives $c^2=a^2+b^2-2ab\cos C$, and the given relation rearranges to $c^2=a^2+b^2-ab$. Comparing, $\cos C=\frac{1}{2}$, so $C=60°$ and the area is $\frac{1}{2}ab\sin 60°=\frac{\sqrt{3}}{4}ab$.

12. B. Put $u=3^x>0$ and $v=3^y>0$, so $u+v^2=10$ and $v+u^2=10$. Subtracting gives $(u-v)-(u^2-v^2)=0$, that is $(u-v)\left[1-(u+v)\right]=0$. If $u=v$ then $u^2+u-10=0$, which has exactly one positive root $u=\frac{-1+\sqrt{41}}{2}$. If instead $u+v=1$ then $u=1-v$ and $v^2-v-9=0$, so $v=\frac{1+\sqrt{37}}{2}>1$ and $u<0$, which is impossible. Exactly one pair (x,y) exists.

13. C. Four lines in general position give $1+\frac{n(n+1)}{2}=1+10=11$ regions.

14. C. The right-hand side is $-\cos^3 x$, so $\sin^3 x=-\cos^3 x$. Here $\cos x\neq 0$, since $\cos x=0$

would force $\sin^3 x = 0$. Hence $\tan^3 x = -1$, so $\tan x = -1$ and $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, with sum $\frac{5\pi}{2}$.

15. C. Note $f(x) = (x-2)^3+3$. Then $f(2-\sqrt[3]{5}) = -5+3 = -2$ and $f(2+\sqrt[3]{5}) = 5+3 = 8$, so the sum is 6.

16. D. Since $f(x) = \left(f(x/2)\right)^2$, f is never negative. From $f(a) = f(a)f(0)$ with $f(a) \neq 0$ we get $f(0) = 1$. Then $f(4) = \left(f(2)\right)^2 = 16$ with $f(2) \geq 0$ gives $f(2) = 4$, and $f(2)f(-2) = f(0) = 1$ gives $f(-2) = \frac{1}{4}$.

17. B. The two probabilities are $\dfrac{\binom{3}{2}\binom{n}{1}}{\binom{n+3}{3}} = \dfrac{3n}{\binom{n+3}{3}}$ and $\dfrac{\binom{3}{1}\binom{n}{2}}{\binom{n+3}{3}} = \dfrac{3n(n-1)/2}{\binom{n+3}{3}}$. Equating and dividing by $3n$ gives $1 = \frac{n-1}{2}$, so $n = 3$.

18. B. Division gives $\dfrac{n^2+7n+22}{n+3} = n+4+\dfrac{10}{n+3}$, so $n+3$ must divide 10. Since $n \geq 1$ forces $n+3 \geq 4$, the only options are $n+3 = 5$ and $n+3 = 10$, giving $n = 2$ and $n = 7$, with sum 9.

19. A. Subtracting x gives $\dfrac{2x-1}{x+2} - x = \dfrac{-(x^2+1)}{x+2} > 0$. The numerator is negative for every real x, so the fraction is positive exactly when $x+2 < 0$, that is $x < -2$. Equality is impossible, so the endpoint is excluded.

20. C. Iterating gives $f_n(x) = \dfrac{x}{\sqrt{1+nx^2}}$, which follows by induction since $f_{n+1}(x) = f(f_n(x))$ replaces $1+nx^2$ by $1+(n+1)x^2$. Hence $f_{15}(1) = \frac{1}{\sqrt{16}} = \frac{1}{4}$.