Test of Mathematics for University Admission

Paper 2

Mock Examination Practice

Instructions for Candidates:

- There are 20 questions in this paper.
- Each question is followed by multiple options.
- Choose the single best option for each question.
- Calculators and dictionaries are NOT permitted.
- Use the blank space on each page for your working out.
- Do not turn over until you are told to do so.

Written by Y&A Tutoring

Cambridge Maths and Economics offer holders, TMUA 8.3 to 9.0 (October 2025)

Tutoring, feedback and further papers: y.a.tutors@gmail.com

Question 1

Consider the statement:

No positive integer N that is less than 20 can be written as the sum of two non-prime integers that are greater than 1.

Which of the following is a counterexample to this statement?

I $N = 8$

II $N = 13$

III $N = 24$

A) none of them

B) I and II only

C) II and III only

D) I and III only

E) I, II and III

Question 2

Five sealed jars, labelled A, B, C, D and E, each contain the same (non-zero) number of £1 coins.

The following statements are attached to the jars:

A: This jar contains less than £4

B: This jar contains £5 or £6

C: This jar contains £1 or £6

D: This jar contains more than £3 and less than £6

E: This jar contains £2 or £3

Exactly one of the jars has a true statement attached to it. Which jar is it?

A) A

B) B

C) C

D) D

E) E

Question 3

A child is chosen at random from a group. Each child is equally likely to be chosen.

Which of the following conditions is/are necessary for the probability that the child has a pet to equal exactly 4/9?

I Exactly 5 children in the group do not have a pet.

II The number of children in the group is divisible by 4.

III The number of children in the group with a pet is divisible by 3.

A) none of them

B) I only

C) II only

D) III only

E) I and II only

F) I and III only

G) II and III only

H) I, II and III

Question 4

Consider the following incorrect attempt at a proof, given that a and b are real numbers such that $a = b$. On which line does the first mistake occur?

A) $a^2 = ab$

B) $a^2 - b^2 = ab - b^2$

C) $(a - b)(a + b) = b(a - b)$

D) $a + b = b$

E) $2b = b$

F) $2 = 1$

Question 5

Consider the following two statements about the polynomial $p(x)$ with exactly two stationary points at $x = a$ and $x = b$ where $a < b$.

P: $p(x)$ has a maximum at $x = a$

Q: $p'(x) < 0$ for $a < x < b$

Which of the following is correct?

A) P is necessary and sufficient for Q

B) P is not necessary and not sufficient for Q

C) P is necessary but not sufficient for Q

D) P is sufficient but not necessary for Q

Question 6

Consider the following attempt at finding the number of solutions to the equation $6\sin x = 7\frac{\sin x}{\cos x}$ for $0 \leq x \leq 360°$.

I $6 = \frac{7}{\cos x}$

II $\cos x = \frac{7}{6}$

III There are no real solutions to this equation.

Which statement describes this attempt?

A) It is completely correct

B) It is incorrect and the first mistake occurs on line I

C) It is incorrect and the first mistake occurs on line II

D) It is incorrect and the first mistake occurs on line III

Question 7

Consider the following two statements about the polynomial $p(x)$:

P: $p(x)$ has at least one real root.

Q: $p(x)$ is a polynomial of order n, where n is an odd integer.

Which of the following is correct?

A) P is necessary and sufficient for Q

B) P is not necessary and not sufficient for Q

C) P is necessary but not sufficient for Q

D) P is sufficient but not necessary for Q

Question 8

Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$:

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$.

Which of the following is true?

A) The condition $p(a)p(b) < 0$ is necessary and sufficient for (*)

B) The condition $p(a)p(b) < 0$ is not necessary and not sufficient for (*)

C) The condition $p(a)p(b) < 0$ is necessary but not sufficient for (*)

D) The condition $p(a)p(b) < 0$ is sufficient but not necessary for (*)

Question 9

The fact that $5 \times 6 = 30$, is a counter example to which of the following statements:

A) the product of any two odd integers is odd

B) if the product of two integers is a multiple of 4 then the integers are not consecutive

C) if the product of two integers is not a multiple of 4 then the integers are not consecutive

D) any even integer can be written as the product of two even integers.

Question 10

A student attempts to answer the following question.

What is the largest number of consecutive odd integers that are all prime?

The student's attempt is as follows:

I There are two consecutive odd integers that are prime (for example: 17, 19).

II Any three consecutive odd integers can be written in the form $n-2, n, n+2$ for some n.

III If n is one more than a multiple of 3, then $n+2$ is a multiple of 3.

IV If n is two more than a multiple of 3, then $n-2$ is a multiple of 3.

V The only other possibility is that n is a multiple of 3.

VI In each case, one of the integers is a multiple of 3, so not prime.

VII Therefore the largest number of consecutive odd integers that are all prime is two.

Which of the following best describes this attempt?

A) It is completely correct.

B) It is incorrect, and the first error is on line I.

C) It is incorrect, and the first error is on line II.

D) It is incorrect, and the first error is on line III.

E) It is incorrect, and the first error is on line IV.

F) It is incorrect, and the first error is on line V.

G) It is incorrect, and the first error is on line VI.

H) It is incorrect, and the first error is on line VII.

Question 11

Consider the two statements:

R: k is an integer multiple of π

S: $\int_0^k \sin 2x \, dx = 0$

Which of the following statements is true?

A) R is necessary and sufficient for S.

B) R is necessary but not sufficient for S.

C) R is sufficient but not necessary for S.

D) R is not necessary and not sufficient for S.

Question 12

Consider the following equation where a is a real number and $a > 1$:

(*) $a^x = x$

Which of the following equations must have the same number of real solutions as (*)?

I $\log_a x = x$

II $a^{2x} = x^2$

III $a^{2x} = 2x$

A) none of them

B) I only

C) II only

D) III only

E) I and II only

F) I and III only

G) II and III only

H) I, II and III

Question 13

Consider the following statement about a pentagon P:

(*) If at least one of the interior angles in P is $108°$, then all the interior angles in P form an arithmetic sequence.

Which of the following is/are true?

I The statement (*)

II The contrapositive of (*)

III The converse of (*)

A) none of them

B) I only

C) II only

D) III only

E) I and II only

F) I and III only

G) II and III only

H) I, II and III

Question 14

Here is an attempt to solve the inequality $x^4 - 2x^2 - 3 < 0$ by completing the square:

I if and only if $x^4 - 2x^2 + 1 < 4$

II if and only if $(x^2 - 1)^2 < 4$

III if and only if $-2 < x^2 - 1 < 2$

IV if and only if $x^2 - 1 < 2$

V if and only if $x^2 < 3$

VI if and only if $-\sqrt{3} < x < \sqrt{3}$

Which of the following statements is true?

A) The argument is completely correct.

B) The first error occurs in line I.

C) The first error occurs in line II.

D) The first error occurs in line III.

E) The first error occurs in line IV.

F) The first error occurs in line V.

G) The first error occurs in line VI.

Question 15

Consider the following theorem:

If $2^k + 1$ is a prime, then k is a power of 2. (*)

Which of the following statements, taken individually, is/are equivalent to (*)?

I If k is a power of 2, then $2^k + 1$ is prime.

II $2^k + 1$ is not prime only if k is not a power of 2.

III A sufficient condition for k to be a power of 2 is that $2^k + 1$ is prime.

A) I only

B) II only

C) III only

D) I and II

E) II and III

F) I and III

G) I, II and III

H) None of them

Question 16

Let x be a real number.

Which one of the following statements is a sufficient condition for exactly three of the other four statements?

A) $x \geq 0$

B) $x = 1$

C) $x = 0$ or $x = 1$

D) $x \geq 0$ or $x \leq 1$

E) $x \geq 0$ and $x \leq 1$

Question 17

The real numbers a, b, c and d satisfy $0 < a < b < c < d$.

Which of the following inequalities must be true?

I $a + c < b + d$

II $d - c > b - a$

III $\frac{b}{c} < \frac{d}{a}$

A) none of them

B) I only

C) II only

D) III only

E) I and III only

F) I, II and III

Question 18

Let $a, b, c > 0$. The equations:

$\log_a b = c$ and $a = b^c$

A) specify a, b and c uniquely

B) specify c uniquely but have infinitely many solutions for a and b

C) specify a and b uniquely but have infinitely many solutions for c

D) have no solutions for a, b and c

E) None of the above

Question 19

Which of the following statements are true?

I There exists a real y such that for all real x, $y > x$

II There exists a real x such that for all real y, $x + y > xy$

III For all real x, there exists real y such that $x - y = xy^2 + 1$

A) none of them

B) I only

C) II only

D) III only

E) II and III only

Question 20

f is a function and a, b are real numbers.

Given that exactly one of the following statements is true, which one is it?

A) $f(a) \geq f(b)$ if and only if $a \geq b$

B) $f(a) \geq f(b)$ only if $a < b$

C) $f(a) < f(b)$ if $a \geq b$

D) $a \geq b$ if $f(a) \geq f(b)$

E) $a < b$ only if $f(a) \geq f(b)$

Answer Key & Brief Hints

1. B. Both $8 = 4 + 4$ and $13 = 4 + 9$ are less than 20 and are sums of two non-prime integers greater than 1, so each contradicts the statement. $N = 24$ does not satisfy the hypothesis, since it is not less than 20, so it cannot be a counterexample.

2. D. Every jar holds the same amount, so test each value in turn. One coin makes A and C true, two or three coins make A and E true, five makes B and D true, six makes B and C true, and seven or more makes every statement false. Only four coins leaves exactly one true statement, namely the one on jar D.

3. A. The group must contain $9k$ children, of whom $4k$ have a pet and $5k$ do not, for some positive integer k. Taking $k = 1$ gives 9 children, 4 with a pet: the total is not divisible by 4 and the number with a pet is not divisible by 3, so II and III are not necessary. Taking $k = 2$ gives 10 children without a pet, so I is not necessary either.

4. D. Lines A to C are valid. The step to $a + b = b$ divides both sides by $a - b$, which is zero because $a = b$.

5. D. If p has a maximum at a then p' changes from positive to negative there, and since there is no further stationary point before b it stays negative on (a, b): P implies Q. The converse fails. Take $p'(x) = -(x-a)^2(x-b)^2$, so that p has exactly two stationary points and $p' < 0$ on (a, b), yet p' is negative on both sides of a, making $x = a$ a stationary point of inflection rather than a maximum. So P is sufficient but not necessary.

6. B. Line I divides both sides by $\sin x$ and so loses the solutions with $\sin x = 0$, namely $x = 0°, 180°, 360°$. Lines II and III follow correctly from line I, so the first error is on line I.

7. C. Every polynomial of odd degree has at least one real root, so Q implies P and P is necessary for Q. P does not imply Q, since $x^2 - 1$ has real roots and even degree.

8. D. If $p(a)p(b) < 0$ then p changes sign on the interval and, being continuous, must vanish somewhere inside it, so the condition is sufficient. It is not necessary: for $p(x) = (x-c)^2$ with $a < c < b$ there is a root in the interval but $p(a)p(b) > 0$.

9. C. The product 30 is not a multiple of 4, yet 5 and 6 are consecutive, so the conclusion of C fails while its hypothesis holds. A does not apply because 5 and 6 are not both odd, B has a false hypothesis since 30 is not a multiple of 4, and D concerns writing an even number as a product of two even numbers, which this product does not address.

10. G. The conclusion is false, since $3, 5, 7$ are three consecutive odd primes. Lines I to V are correct. Line VI is the first error: a multiple of 3 can be prime, namely 3 itself.

11. A. $\displaystyle\int_0^k \sin 2x \, dx = \tfrac{1}{2}(1 - \cos 2k)$, which vanishes exactly when $\cos 2k = 1$, that is $2k = 2m\pi$ and so $k = m\pi$ for an integer m. The two statements are therefore equivalent.

12. F. Statement I is the same equation: $\log_a x = x$ rearranges to $x = a^x$. Statement III is $(*)$ after the substitution $y = 2x$, which is a bijection of the reals, so the number of solutions is unchanged. Statement II is different: $a^{2x} = x^2$ gives $(a^x)^2 = x^2$, hence $a^x = x$ or $a^x = -x$, and the second equation contributes exactly one further solution, which is negative and so cannot coincide with a solution of $a^x = x$.

13. D. The interior angles of a pentagon sum to $540°$, so five angles in arithmetic progression have middle term $108°$: the converse III is true. The statement itself is false, since the angles $108°, 100°, 110°, 120°, 102°$ include $108°$ but are not in arithmetic progression, and the contrapositive II is logically equivalent to the statement, so it is false as well.

14. A. Every step is reversible. In particular line IV is valid because $x^2-1 \geq -1 > -2$ for all real x, so the left-hand inequality in line III is automatically satisfied and discarding it loses nothing.

15. C. The theorem says that 2^k+1 being prime is enough to force k to be a power of 2, which is exactly what III states. I is the converse. In II, "A only if B" means A implies B, so II reads: 2^k+1 not prime implies k not a power of 2, whose contrapositive is I. Neither I nor II is equivalent to the theorem.

16. C. If $x=0$ or $x=1$ then A, D and E all hold, while B need not, giving exactly three of the other four. For comparison, B implies all four of the others, A implies only D, D is always true and so implies none of the others, and E implies only A and D.

17. E. Adding $a<b$ to $c<d$ gives I. For III, $\frac{b}{c} < 1 < \frac{d}{a}$ since $b<c$ and $a<d$. Statement II fails, for example $a=1$, $b=10$, $c=11$, $d=12$ gives $d-c=1$ and $b-a=9$.

18. B. The first equation means $b=a^c$, and substituting into $a=b^c$ gives $a=a^{c^2}$. Since $\log_a$ requires $a \neq 1$, this forces $c^2=1$ and hence $c=1$, as $c>0$. Then $b=a$, and every $a>0$ with $a \neq 1$ works, so c is determined uniquely while a and b are not.

19. E. I is false, since the real numbers have no greatest element. II is true: take $x=1$, and then $1+y>y$ for every real y. III is true: the equation rearranges to $xy^2-y+(1-x)=0$, which for $x=0$ gives $y=-1$, and for $x \neq 0$ is a quadratic in y with discriminant $1-4x(1-x)=(2x-1)^2 \geq 0$.

20. E. Read each statement as a claim holding for all a and b. Taking $a=b$ makes B and C false. If D holds then $a<b$ implies $f(a)<f(b)$, so f is strictly increasing, which makes A true as well; and A implies D. So neither A nor D can be the only true statement. That leaves E, and a constant function confirms it: E holds while A, B, C and D all fail.