Mock TMUA Set C: Paper 2

20 questions
75 minutes
No calculator allowed

1.

Given that $\frac{dy}{dx} = 6x^2 - \frac{4x-3}{x^4}$, $x \neq 0$ and $y = 6$ when $x = 1$, find y in terms of x.

A $y = 12x + 12x^{-2} - 12x^{-3} - 6$

B $y = 12x + 12x^{-4} - 12x^{-5} - 6$

C $y = 2x^3 + x^{-4} - x^{-3} + 4$

D $y = 2x^3 + 2x^{-2} - x^{-3} + 3$

E $y = 2x^3 + 2x^{-2} + x^{-3} + 1$

2.

Find the complete set of values of the real constant p for which the expression

$$x^2 - 2x + px - 2 + p$$

is positive for all real values of x .

A $4 - 2\sqrt{5} < p < 4 + 2\sqrt{5}$

B $p < 4 - 2\sqrt{5}$ or $p > 4 + 2\sqrt{5}$

C $-2 < p < 2$

D $p < -2$ or $p > 2$

E $2 < p < 6$

F $p < 2$ or $p > 6$

3.

The real numbers a, b, and c are non-zero and $a \leq b$.

Which of the following statements are necessarily true?

I $\quad \dfrac{1}{a} \geq \dfrac{1}{b}$

II $\quad 2^a \leq 2^b$

III $\quad ac \leq bc$

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

4.

A bag only contains $2n$ blue balls and n red balls. All the balls are identical except in colour. One ball is randomly selected and not replaced. A second ball is then randomly selected. What is the probability that at least one of the selected balls is red?

A $\quad \dfrac{n(n-1)}{3(3n-1)}$

B $\quad \dfrac{3n-1}{3(3n-1)}$

C $\quad \dfrac{4n-2}{3(3n-1)}$

D $\quad \dfrac{2n(n-1)}{3(3n-1)}$

E $\quad \dfrac{5n-1}{3(3n-1)}$

5.

A student attempts to prove the following statement.

> Consider the integers a and b, where a has remainder 1 when divided by 3, and b has remainder 2 when divided by 3.
>
> Then $a + b$ is always divisible by 3.

Consider the following attempt:

Let	$a = 3n + 1$ and $b = 3n + 2$	(I)
then	$a + b = 3n + 1 + 3n + 2$	(II)
so	$a + b = 6n + 3$	(III)
so	$a + b = 3(2n + 1)$	(IV)
therefore	$a + b$ is always divisible by 3.	(V)

Which of the following best describes this proof?

A The statement is true and the proof is completely correct.

B The statement is true but there is an error in the proof in line (I)

C The statement is true but there is an error in the proof in line (II)

D The statement is not true and there is an error in the proof in line (III)

E The statement is not true and there is an error in the proof in line (IV)

F The statement is not true and there is an error in the proof in line (V)

6.

This question uses radians.

Find the number of distinct values of x that satisfy the equation

$$2(2 - x)(x - 1) = 1 - sin\pi x$$

A 0 B 1 C 2 D 3 E 4

7.

Consider the statement:

(*) Every prime number n can be written as the sum of 2 square numbers.

How many counterexamples to (*) are there in the range $0 < n < 40$

A 2
B 3
C 4
D 5
E 6

8.

The notation $\lfloor x \rfloor$ means the greatest integer less than or equal to x.

For example $\lfloor 0.8 \rfloor = 0$ $\lfloor 2 \rfloor = 2$ $\lfloor \sqrt{12} \rfloor = 3$

Evaluate the integral $\int_{\frac{\pi}{2}}^{\pi} x \lfloor x \rfloor \, dx$

A $\frac{26}{3}$ B $\frac{11\pi^2}{8} - \frac{13}{2}$ C $\frac{11\pi^2}{8}$ D $\frac{7\pi^3}{24}$ E $\frac{7\pi^3}{24} - \frac{13}{8}$

9.

A locked box has two levers, A and B, which can be positioned either left or right at any particular time. It is known that if lever A is left or lever B is right, then the box is unlocked.

Which of the following statements must be true?

A If the box is unlocked then lever A is left or lever B is right
B If the box is locked then lever A is left and lever B is right
C If the box is unlocked then lever A is left and lever B is right
D If the box is locked then lever A is right or lever B is left
E If the box is unlocked then lever A is right or lever B is left
F If the box is locked then lever A is right and lever B is left

10.

A triangle ABC is to be drawn with the following measurements.

$AB = 3a$ $BC = b$ angle $BAC = 45°$ where a and b are constants

For which values of b can two distinct triangles ABC be drawn ?

A $b < 3a$

B $a < b < 3a$

C $\frac{3}{2}\sqrt{2}\, a < b < 3a$

D $b > \sqrt{2}\, a$

E $b > \frac{3}{2}\sqrt{2}\, a$

11.

Consider the statement: $f(x) > g(x)$ for all real values of $x > a$

Which one of the following is a negation of this statement?

A $f(x) \leq g(x)$ for at least one real value of $x > a$

B $f(x) \leq g(x)$ for all real values of $x > a$

C $f(x) \leq g(x)$ for at least one real value of $x \leq a$

D $f(x) \leq g(x)$ for no real values of $x > a$

E $f(x) > g(x)$ for at least one real value of $x \leq a$

F $f(x) > g(x)$ for at least one real value of $x > a$

G $f(x) > g(x)$ for all real values of $x \leq a$

H $f(x) > g(x)$ for no real values of $x > a$

12.

The function $F(n)$ is defined for all positive integers as follows:

$$F(1) = 1 \qquad \text{and for all } n \geq 2$$

$$F(n) = F(n-1) + 5 \qquad \text{if 5 divides } n \text{ but 2 does not divide } n$$

$$F(n) = F(n-1) + 2 \qquad \text{if 2 divides } n \text{ but 5 does not divide } n$$

$$F(n) = F(n-1) - 1 \qquad \text{if 2 and 5 both divide } n$$

$$F(n) = F(n-1) \qquad \text{if neither 2 nor 5 divides } n$$

The value of $F(301)$ is equal to

A 300 B 301 C 361 D 363 E 372

13.

Consider the following statements for real values of x.

P: $\sqrt{x+12} \geq x$

Q: $-3 \leq x \leq 4$

Which one of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **sufficient** but **not necessary** for Q

D P is **necessary** but **not sufficient** for Q

14.

A list consists of k integers, and the mean of these is calculated to be m .

When an integer a is added to this list, the mean decreases by 1.

When a further integer b is added to the new list, the mean decreases again by another 1.

Which one of the following statements is true?

A $m = k + 4$

B $k < m$

C $a + b = 2(m - k)$

D $a(m - 1) = b(m - 2)$

E $a - b = 2$

15.

Consider the following statement:

If $f'(x) > 0$ *for all real x where it is defined,* *then* $f(x + 1) > f(x)$ *for all real x*

Which function provides a counterexample:

A $f(x) = 4^x$ B $f(x) = 4x^2 + 1$ C $f(x) = 4x^3$

D $f(x) = \dfrac{4 - x}{x}$ E $f(x) = \dfrac{x - 1}{4x}$

16.

Given that $(x^2 + x + 2)^{24} = a_0 + a_1x + \ldots + a_{48}x^{48}$

Find the value of $a_0 + a_2 + a_4 + \ldots + a_{48}$

A $2^{24}(2^{22} + 1)$

B $2^{24}(2^{22} - 1)$

C $2^{23}(2^{24} - 1)$

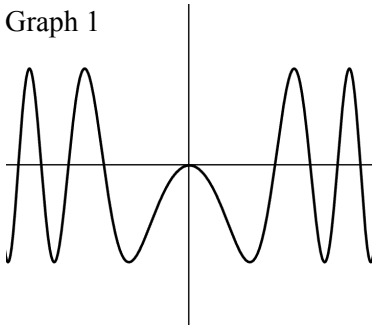
D $2^{23}(2^{24} + 1)$

E $2^{23}(2^{25} + 1)$

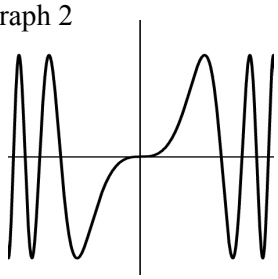
17.

Graphs 1 and 2 represent functions of the form $y = a\sin(x^b)$ where a, b are non-zero integers

Graph 1



Graph 2



Which of the following statements are necessarily true?

I In Graph 1, $a < 0$ and b is even

II In Graph 2, $a > 0$ and $b < 0$

III $b > 0$ in both Graph 1 and Graph 2

A none of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II and III

18.

$P(x)$ and $Q(x)$ are defined as follows: $P(x) = 3^{(2x-1)} - 3^{(x+1)} + 3$ $Q(x) = 3^x - 3$.

What is the largest value of x such that $P(x)$ and $Q(x)$ are in the ratio $5 : 3$ respectively.

A $log_3 2$ B $log_3 5$ C $log_3 12$ D 3 E 5

19.

A curve has the equation $y = ax^3 + bx^2 + c$

The curve has a maximum stationary point at $x = 0$ and a minimum stationary point in the 4th quadrant (the region where $x > 0$ and $y < 0$).

Which of the following set of conditions is **sufficient** to ensure this?

A $a < 0, b < 0, c < 0$

B $a > 0, b < 0, c < 0$

C $a < 0, b > 0, c < 0$

D $a > 0, b < 0, c > 0$

E $a > 0, b > 0, c > 0$

F None of the above

20.

f is a function and *a is* a real number.
Given that exactly one of the following statements is true, which one is it?

A $a \leq 0$ **only if** $f(a) \leq 0$

B $f(a) > 0$ **if** $a > 0$

C $f(a) > 0$ is **sufficient** for $a > 0$

D $f(a) \leq 0$ is **necessary** for $a \leq 0$

E **If** $f(a) > 0$ **then** $a > 0$

F $a > 0$ **if** $f(a) > 0$