

Mock TMUA Set D: Paper 1

20 questions

75 minutes

No calculator allowed

1.

Given that $2x^2 + 11x + 5$ is a factor of $ax^3 - 17x^2 + bx - 15$

What is the value of $a + b$?

- A -40 B -28 C -4 D 4 E 28 F 40

2.

The point $(-1, -1)$ lies on the curve C whose gradient function is given by

$$\frac{dy}{dx} = \frac{5x^3 - 6}{x^3} \quad x \neq 0$$

Find an equation for C .

- A $y = 5x^4 - 6x - 12$
- B $y = 5x + \frac{3}{x^2} + 1$
- C $y = 5x + \frac{3}{2x^4} + \frac{7}{2}$
- D $y = 15x^2 + \frac{18}{x^2} - 34$
- E $y = 5x - \frac{3}{x^2} + 7$

3.

A circle of radius 1cm is inscribed inside an equilateral triangle. What is the area of the triangle?

- A $\frac{\sqrt{3}}{3}$ B 3 C $\pi + \frac{\sqrt{3}}{3}$ D $3\sqrt{3}$ E $6\sqrt{3}$

4.

S is the complete set of values of x which satisfy **both** the inequalities

$$x^2 - x - 6 < 0 \quad \text{and} \quad 4 - 3x > 1$$

The set S can also be represented as a single inequality.

Which of the following single inequalities represents the set S ?

A $x^2 + x - 2 < 0$

B $x^2 + x - 2 > 0$

C $x^2 - 4x + 3 < 0$

D $x^2 - 4x + 3 > 0$

E $x^2 + 2x < 0$

F $x < 1$

5.

A cylinder has its radius reduced by 20%.

In order to keep the volume of the cylinder the same, the height needs to be increased by:

A 20%

B 40%

C 56.25%

D 62.5%

E 64%

6.

An arithmetic progression and a convergent geometric progression each have first term 1.

The sum of the second terms of the two progressions is $\frac{5}{4}$.

The sum of the third terms of the two progressions is $\frac{3}{4}$.

What is the sum to infinity of the geometric progression?

A -2 B $-\frac{3}{2}$ C $-\frac{1}{2}$ D $\frac{1}{2}$ E 2

7.

How many solutions does the following equation have in the range $0 \leq x < 2\pi$

$$\cos(\sin x) = \frac{1}{2}$$

- A 0
- B 1
- C 2
- D 3
- E infinitely many

8.

Find the area enclosed by the graph of $|2x| + |y| = 2$

- A $2\sqrt{5}$
- B 4
- C $2\sqrt{2}$
- D $\frac{5}{2}$
- E 2

9.

Find the maximum value of the function $\frac{1}{9^x - 6(3^x) + 11}$

- A $\frac{1}{11}$
- B $\frac{1}{3}$
- C $\frac{4}{11}$
- D $\frac{1}{2}$
- E $\frac{11}{2}$

10.

The graph of $y = x^2 - 4$ has a series of transformations applied, resulting in the graph of $y = x^2 - 2x$

Which of the following could be the series of transformations?

- A a translation two units left followed by a reflection in the y-axis
- B a translation one unit left followed by a reflection in the y-axis
- C a translation one unit right followed by a translation three units up
- D a translation one unit right followed by a translation four units up
- E a reflection in the y-axis followed by a translation four units up

11.

What is the sum of the solutions to the equation $9\log_2(4x^2) = (2\log_2 4x)^2$ for $x > 0$

- A $\frac{1}{2}$ B 1 C $\frac{1 + \sqrt{2}}{2}$ D $\frac{5}{2}$ E $\frac{4 + \sqrt{2}}{2}$

12.

A triangle ABC is to be drawn with the following measurements.

$$AB = \sqrt{7}a \quad BC = 2a \quad \text{angle } BAC = \theta^\circ$$

Of the two possible triangles that could be drawn, the area of the larger triangle and the area of the smaller triangle are in the ratio 3:1

What is the value of $\cos\theta$?

- A $\frac{2}{\sqrt{7}}$ B $\frac{\sqrt{3}}{\sqrt{7}}$ C $\frac{4}{\sqrt{7}}$ D $\frac{3}{7}$ E $\frac{4}{7}$

13.

C_1 and C_2 are circles defined by the equations

$$(x - 12)^2 + y^2 = 36 \quad \text{and} \quad (x + 13)^2 + y^2 = 81 .$$

Find the length of the shortest line segment PQ which is tangent to C_1 at P and tangent to C_2 at Q .

- A 15 B 16 C 18 D 20 E $\sqrt{616}$

14.

In the simplified expansion of $(2 + 5x)^9$.

How many of the terms have a coefficient that is divisible by 60?

- A 6 B 7 C 8 D 9 E 10

15.

It is given that $y = (1 - 2\sin x) \cos 2x$ for $0 < x < 2\pi$.

The fraction of the interval $0 < x < 2\pi$ for which $y < 0$ is

- A $\frac{1}{6}$ B $\frac{1}{4}$ C $\frac{1}{3}$ D $\frac{1}{2}$ E $\frac{2}{3}$

16.

How many real roots does the equation $y = 3x^4 - 16x^3 + 18x^2 - 5$ have?

- A 1 B 2 C 3 D 4 E 5

17.

Given that the real numbers x and y satisfy the equations $4^x + 4^y = 10$
and $2(4^x) + 4^{2y} = 20$

What is the value of $x + y$?

- A $\log_4 10$ B 2 C 4 D $4 + \log_4 10$ E 10

18.

It is given that $f(x) = x^2 - 4x + 2$.

The curves $y = f(kx)$ and $y = f(c - x)$ have the same minimum point, where $k > 0$ and $c > 0$.

Which of the following is an expression for k in terms of c ?

- A $k = \frac{c+2}{2}$ B $k = \frac{2}{c-2}$ C $k = \frac{2}{c}$ D $k = \frac{c-4}{4}$ E $k = \frac{c}{4}$

19.

Given that $x^2 + y^2 = 1$, what is the greatest possible value of $2x + 3y$

A $\sqrt{13}$

B $\sqrt{10}$

C $\sqrt{7}$

D $\frac{5\sqrt{2}}{2}$

E 3

20.

The sum of the maximum and minimum values of the function

$$f(x) = (2a)^{\sin x} \quad \text{where } a > 0 \text{ and } x \text{ is real, is 4.}$$

Find the sum of the possible values of a .

A $2 - \sqrt{3}$

B $\sqrt{3}$

C $1 + \frac{1}{2}\sqrt{3}$

D 2

E $2 + \sqrt{3}$