TMUA Mock Paper 1

TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

PAPER 1

75 MINUTES

By sed :)

For questions or corrections, please contact me at @just.sed on Discord. Good luck!
Answer key and a QR code to the full solutions are on the last 2 pages.

(Note: A small error was present in Question 13, which has now been fixed.)

1. The full range of values of x for which

$$x^3 - 2x^2 - x + 2 > 0$$

is:

A. $x < -1$ **or** $1 < x < 2$;

B. $-2 < x < -1$ **or** $x > 2$;

C. $-1 < x < 1$ **or** $x > 2$;

D. $-1 < x < 2$;

E. $\dfrac{-1-\sqrt{17}}{4} < x < \dfrac{-1+\sqrt{17}}{4}$;

F. $x < \dfrac{-1-\sqrt{17}}{4}$ **or** $x > \dfrac{-1+\sqrt{17}}{4}$.

2. The area bounded within

$$x^2 + y^2 = 1$$

and above

$$y = -x + 1$$

is:

A. $\frac{\pi}{2}$;

B. $\sqrt{2} + \frac{\pi}{4}$;

C. $\sqrt{2}$;

D. $\frac{\pi}{4} - \frac{1}{2}$;

E. $1 - \frac{\pi}{4}$;

F. $\frac{3\pi}{4} + \frac{1}{2}$.

3. A regular hexagon H has centre O. Let L be a line through O which rotates clockwise continuously.

Denote θ as the total clockwise angle rotated by L.

Denote P, Q as the two points at which L intersects H.

It follows that the graph of the length of PQ against θ has period:

A. $15°$;

B. $30°$;

C. $60°$;

D. $120°$;

E. $180°$;

F. $360°$.

4. Let L_1, L_2 be the 2 tangent lines of

$$(x-1)^2 + (y-1)^2 = 4$$

which pass through $(-2, 4)$. The acute angle between L_1 and L_2 is 2θ.

It follows that $\tan\theta$ is:

A. $\dfrac{\sqrt{18}}{9}$;

B. $\dfrac{\sqrt{14}}{7}$;

C. $2\sqrt{2}$;

D. $\dfrac{\sqrt{22}}{11}$;

E. $\dfrac{2\sqrt{14}}{5}$.

5. Consider the sequence:

- $a_1 = 2$
- $a_n = \dfrac{a_{n-1}+1}{a_{n-1}-1}$ for $n > 1$.

The least value of n for which $\left(\sum_{i=1}^{n} a_i\right) > 5000$ is:

A. 6;

B. 1000;

C. 1001;

D. 2000;

E. 2001;

F. No such n exists.

6. a, b are constants with $a > b > 0$. When $(a + bx)^3$ is expanded,

- the sum of the coefficients of x^i $(0 \leqslant i \leqslant 3)$ is 27;
- the product of the coefficients of x^i is 9.

It follows that $2a + b$ is:

A. $\dfrac{9+\sqrt{5}}{2}$;

B. $\dfrac{9-\sqrt{5}}{2}$;

C. $\dfrac{9+\sqrt{13}}{2}$;

D. $\dfrac{9-\sqrt{13}}{2}$;

E. $\dfrac{27+\sqrt{77}}{2}$;

F. $\dfrac{27-\sqrt{77}}{2}$.

7. A bacteria has initial nonzero population B. After each hour, the bacteria population has a probability p of doubling, and a probability $(1 - p)$ of halving.

Given that the expected population after 2 hours is $\frac{9}{16}B$, p equals:

A. $\frac{1}{24}$;

B. $\frac{1}{6}$;

C. $\frac{1}{4}$;

D. $\frac{-7+\sqrt{69}}{26}$;

E. $\frac{-8+3\sqrt{11}}{14}$.

8. The full range of values of k for which the graph of

$$y = k \sin x + (1 - k)$$

intersects the graph of

$$y = \sin x + (4 + k)$$

is:

A. $-6 \leqslant k \leqslant -\frac{4}{3}$;

B. $k \leqslant -6$ **or** $k \geqslant -\frac{4}{3}$;

C. $-4 \leqslant k \leqslant -\frac{2}{3}$;

D. $k \leqslant -4$ **or** $k \geqslant -\frac{2}{3}$;

E. $-2 \leqslant k \leqslant -\frac{4}{3}$;

F. $k \leqslant -2$ **or** $k \geqslant -\frac{4}{3}$;

G. for all values of k;

H. for no values of k.

9. The area of the triangle formed by the stationary points of

$$y = x^4 - 8x^2 + 17$$

is:

A. 16;

B. $16\sqrt{2}$;

C. 32;

D. 34;

E. 64.

10. You are given that
$$\int_0^{\pi} \sin^5 x \; dx = \frac{16}{15}.$$
Define the total area bounded between the curves
$$\begin{cases} y = x \\ y = 2\sin^5 x + x \end{cases}$$
for $0 \leqslant x \leqslant a$ as $F(a)$.

It follows that $F(a) = 16$ when a equals:

A. $\dfrac{16\pi}{15}$;

B. $\dfrac{8\pi}{15}$;

C. 8π;

D. 16π;

E. 15π;

F. $\dfrac{15\pi}{2}$;

G. none of the values above.

11. Let $T = (4, 6)$ be a point on the circle

$$C : (x - 3)^2 + (y - 4)^2 = 5.$$

Let $P = (p, q)$. The circle centred at P passing through T has area $\pi(p^2 + q^2)$ and is tangent to C at T.

It follows that P is:

A. $(2, 3)$;

B. $\left(\frac{41}{22}, \frac{19}{11}\right)$;

C. $\left(\frac{17}{4}, \frac{3}{2}\right)$;

D. $\left(-\frac{7}{8}, -\frac{15}{4}\right)$;

E. $\left(\frac{19}{8}, \frac{11}{4}\right)$.

12. When $(ax-2)^2$ is divided by $(x-a)$, the remainder is a^2.

The sum of the possible remainder values when $(ax+2)^2$ is divided by $(x+a)$ is:

A. -3;

B. -1;

C. 1;

D. 5;

E. 10;

F. 45.

13. The equations

$$\begin{cases} 2^x + 2^{y+1} = p \\ 2^y + 2^{x+1} = q \end{cases}$$

have a solution pair (x, y) precisely for:

A. $\dfrac{1}{2} < \dfrac{p}{q} < 2$ where $p, q > 0$;

B. $\dfrac{1}{2} \leqslant \dfrac{p}{q} \leqslant 2$ where $p, q > 0$;

C. $\dfrac{1}{2} < \left|\dfrac{p}{q}\right| < 2$;

D. all values of p and q where $p, q \neq 0$;

E. all values of p and q where $p, q > 0$;

F. all values of p and q.

14. Which of the following integrals has the largest value? Assume trigonometric functions are in radians.

A. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \; dx;$

B. $\displaystyle\int_{0}^{\pi} \sin^4 x \; dx;$

C. $\displaystyle\int_{0}^{\pi} |2\sin(2x)| \; dx;$

D. $\displaystyle\int_{0}^{2} \sqrt{4-x^2} \; dx;$

E. $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} -\log_2(\cos x) \; dx.$

15. The function $f(x)$ satisfies:

- $f(x) = 3x,\ 0 \leqslant x \leqslant 1$
- $f(x-1) = f(1-x) = f(1+x)$ for all x

The number of solutions to $f(x) = \dfrac{x}{3} - 1$ is:

A. 1;

B. 3;

C. 5;

D. 7;

E. 9;

F. 11.

16. The tangent to

$$y = x^3 - 3x$$

at $x = a$ (where $a > 0$) intersects the curve again such that the area bounded between the curve and the tangent is $\dfrac{64}{3}$.

It follows that a equals:

A. $\dfrac{4}{15^{\frac{1}{4}}}$;

B. $\dfrac{4}{21^{\frac{1}{4}}}$;

C. $\dfrac{4}{33^{\frac{1}{4}}}$;

D. $\dfrac{4}{57^{\frac{1}{4}}}$;

E. 1;

F. $\dfrac{4}{3}$.

17. In triangle ABC,

- $\angle BAC = 30°$;
- $AC = 12$;
- $\frac{6}{7} < \sin(\angle ABC) \leqslant 1$.

Denote S as the area of ABC. It follows that the range of S is:

A. $18\sqrt{3} - \frac{3\sqrt{13}}{2} < S < 18\sqrt{3} + \frac{3\sqrt{13}}{2}$;

B. $18\sqrt{3} - 3\sqrt{13} < S < 18\sqrt{3} + 3\sqrt{13}$;

C. $0 < S < 18\sqrt{3} + 9\sqrt{13}$;

D. $18\sqrt{3} < S < 18\sqrt{3} + 3\sqrt{13}$;

E. $18\sqrt{3} - \frac{36}{\sqrt{13}} < S < 18\sqrt{3} + \frac{36}{\sqrt{13}}$.

18. The function $f(n)$ satisfies:

- $f(1) = 1$;
- $f(2n) = f(n)$ for all $n \geqslant 1$;
- $f(2n+1) = \dfrac{f(n)}{2}$ for all $n \geqslant 1$.

It follows that $\left(\sum_{i=1}^{2^n} f(i)\right)$ is equal to:

A. $\dfrac{2^n + n + 1}{2}$;

B. $\dfrac{2^{n-1} + n}{2}$;

C. $\dfrac{n^2 + n + 2}{2}$;

D $2\left(\left(\dfrac{3}{2}\right)^n - 1\right)$;

E. $2\left(\dfrac{3}{2}\right)^n - 1$.

19. The minimum distance between the curves

$$y = x^2 + 1$$

and

$$x = y^2 + 1$$

is:

A. $\dfrac{3\sqrt{17}}{5}$;

B. $\sqrt{2}$;

C. $\dfrac{3\sqrt{2}}{8}$;

D. $\dfrac{3\sqrt{2}}{4}$;

E. $\dfrac{7\sqrt{2}}{8}$;

F. 0.

20. For all integers $n \geqslant 1$, define the function $f(x)$ as

$$f(x) = (-1)^{n+1}\sqrt{2^{2-2n} - \left(x - 4 + \frac{6}{2^n}\right)^2}$$

for $4 - 2^{3-n} \leqslant x < 4 - 2^{2-n}$ only.

It follows that the total length of the curve $f(x)$ is:

A. $\dfrac{3\pi}{5}$;

B. $\dfrac{2\pi}{3}$;

C. $\dfrac{3\pi}{2}$;

D. π;

E. 2π;

F. 4π;

G. 4.

ANSWER KEY

Question	Answer
1	C
2	D
3	C
4	B
5	E
6	A
7	B
8	C
9	C
10	F
11	E
12	D
13	A
14	C
15	E
16	F
17	B
18	E
19	D
20	E

Full Solutions