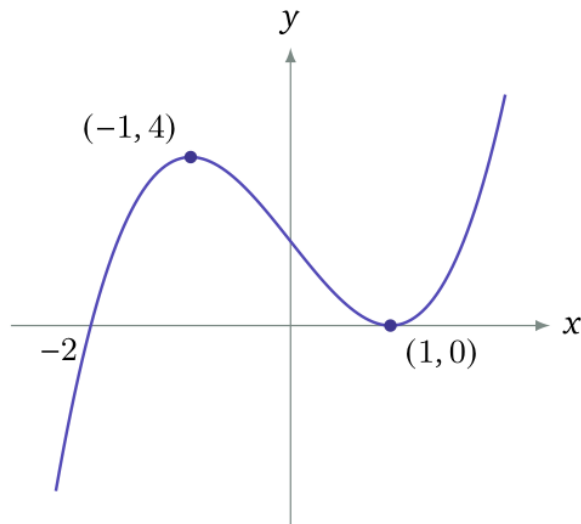


**TMUA Challenge Paper 1 Tmua-p1-challenge-2**

20 questions. Recreated from the publicly available paper.

**Question 1**

The diagram shows the graph of  $y = f(x)$ , where  $f(x) = (x - 1)^2(x + 2)$ .

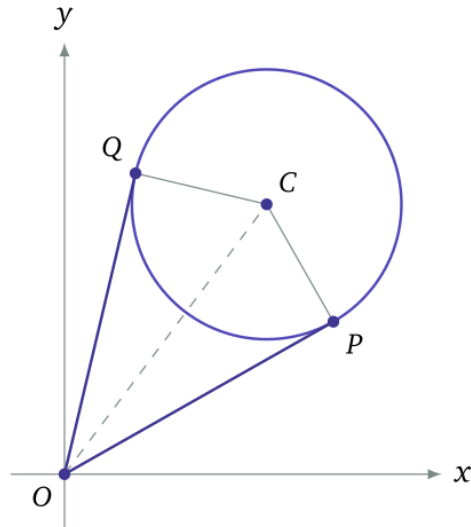


Find the complete set of values of  $k$  for which the equation  $|f(x)| = k$  has exactly four distinct real solutions.

- A.  $k = 0$
- B.  $0 \leq k < 4$
- C.  $0 < k \leq 4$
- D.  $k > 4$
- E.  $0 < k < 2$
- F.  $0 < k < 4$
- G.  $2 < k < 4$
- H. there is no such  $k$

**Question 2**

The diagram shows the two tangents drawn from the origin  $O$  to the circle of radius 2 with centre  $C = (3, 4)$ , touching it at  $P$  and  $Q$ .



What is the area of the quadrilateral  $OPCQ$ ?

A.  $\sqrt{21}$

B.  $2\sqrt{5}$

C.  $2\sqrt{21}$

D.  $4\sqrt{21}$

E. 5

F. 10

G.  $\frac{21}{2}$

H.  $4\sqrt{5}$

**Question 3**

Let  $n$  be an integer greater than 6 for which  $n - 1$  and  $n + 1$  are both prime. What is the largest integer that must be a factor of  $n^2(n^2 + 16)$ ?

A.12

B.48

C.120

D.144

E.240

F.360

G.720

H.1440

**Question 4**

How many solutions does  $\sin 3x = \cos 2x$  have for  $0 \leq x \leq 2\pi$ ?

A.4

B.5

C.6

D.7

E.8

F.9

G.10

H.12

**Question 5**

Evaluate  $\int_0^1 x(1-x)^7 \, dx$ .

A.  $\frac{1}{72}$

B.  $\frac{1}{80}$

C.  $\frac{1}{64}$

D.  $\frac{1}{63}$

E.  $\frac{1}{56}$

F.  $\frac{1}{54}$

G.  $\frac{1}{9}$

H.  $\frac{1}{8}$

**Question 6**

How many distinct tangents to the curve  $y = x^3 - 3x$  pass through the point  $(0, 2)$ ?

A.0

B.1

C.2

D.3

E.4

F.5

G.6

H.infinitely many

**Question 7**

How many ordered pairs  $(a, b)$  of non-negative integers satisfy  $\sqrt{a} + \sqrt{b} = \sqrt{2009}$  ?

A.0

B.1

C.2

D.4

E.7

F.8

G.14

H.infinitely many

**Question 8**

The real numbers  $x$ ,  $y$  and  $z$  satisfy  $x^3 + y^3 + z^3 - 3xyz = 1$ . What is the smallest possible value of  $x^2 + y^2 + z^2$ ?

A.  $\frac{1}{3}$

B.  $\frac{1}{2}$

C.  $\frac{2}{3}$

D. 1

E.  $\frac{3}{2}$

F. 2

G. 3

H. there is no smallest value

**Question 9**

The line  $y = mx + 1$  is a tangent to the circle  $x^2 + y^2 - 6x + 8 = 0$ . What is the sum of the possible values of  $m$ ?

A.  $-\frac{3}{2}$

B.  $-1$

C.  $-\frac{3}{4}$

D.  $-\frac{1}{2}$

E.  $0$

F.  $\frac{3}{4}$

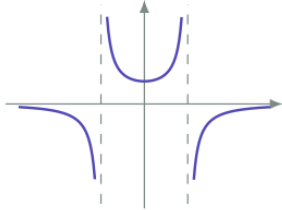
G.  $1$

H.  $\frac{3}{2}$

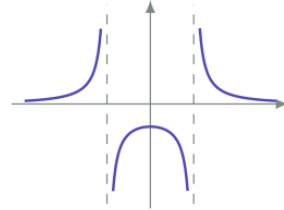
**Question 10**

Which sketch shows the graph of  $y = \frac{1}{x^2 - 1}$  ?

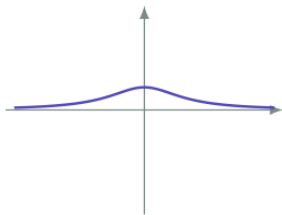
A.



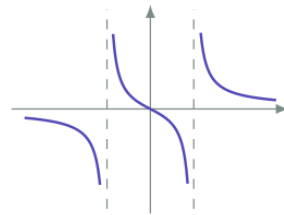
B.



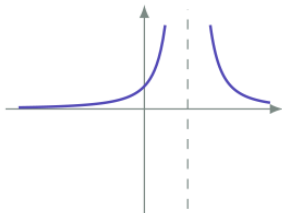
C.



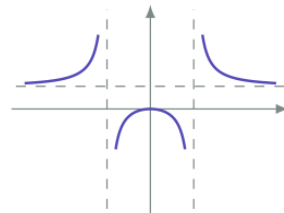
D.



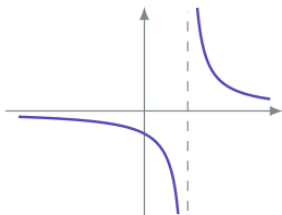
E.



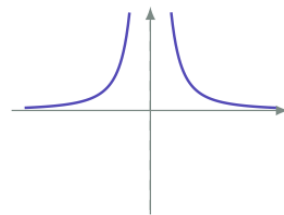
F.



G.



H.



**Question 11**

Find the set of real values of  $x$  for which  $|x - 2| < |2x + 1|$ .

A.  $-3 < x < \frac{1}{3}$

B.  $x < -\frac{1}{3}$  or  $x > 3$

C.  $\frac{1}{3} < x < 3$

D.  $x < -3$

E.  $x > \frac{1}{3}$

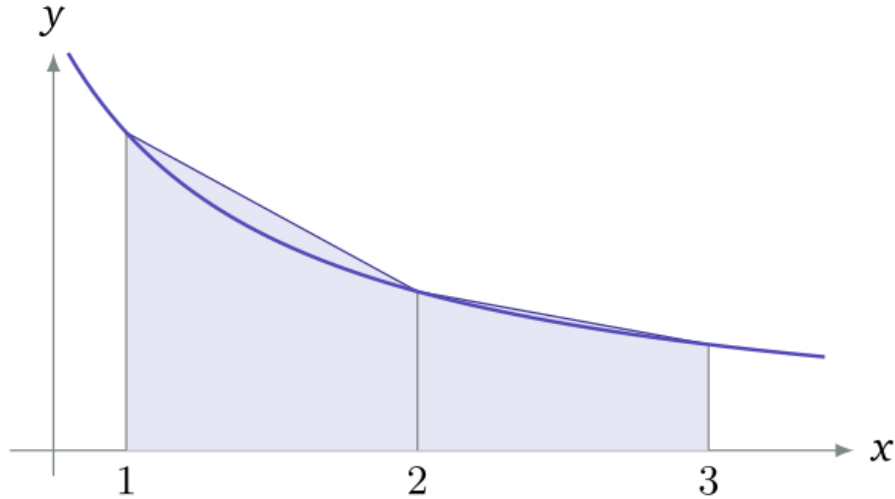
F.  $-\frac{1}{3} < x < 3$

G.  $x < -3$  or  $x > -\frac{1}{3}$

H.  $x < -3$  or  $x > \frac{1}{3}$

**Question 12**

The trapezium rule with two strips of equal width is used to estimate  $\int_1^3 \frac{1}{x} dx$ , as shown.

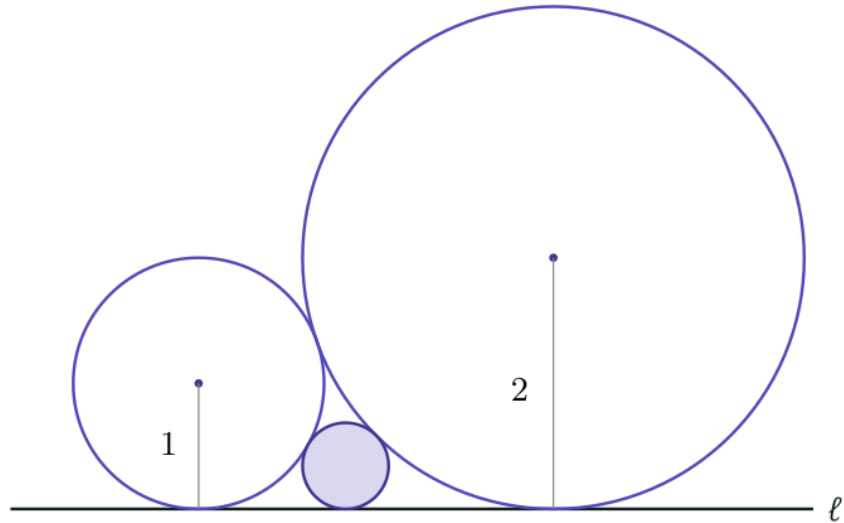


Which statement is correct?

- A. The estimate is  $\frac{7}{12}$ , and it is an underestimate.
- B. The estimate is  $\frac{7}{12}$ , and it is an overestimate.
- C. The estimate is  $\frac{4}{3}$ , and it is an overestimate.
- D. The estimate is  $\frac{7}{6}$ , and it is an overestimate.
- E. The estimate is  $\frac{7}{6}$ , and it is an underestimate.
- F. The estimate is  $\frac{7}{6}$ , and it is exact.
- G. The estimate is  $\frac{11}{6}$ , and it is an overestimate.
- H. The estimate is  $\frac{4}{3}$ , and it is an underestimate.

**Question 13**

Two circles, of radii 1 and 2, touch each other and both touch the line  $\ell$ . A third circle lies in the region between them, touching both circles and  $\ell$ , as shown. What is its radius?



A.  $\frac{1}{3}$

B.  $\frac{2}{5}$

C.  $6 - 4\sqrt{2}$

D.  $3 - 2\sqrt{2}$

E.  $\frac{2}{9}$

F.  $\sqrt{2} - 1$

G.  $\frac{1}{2}$

H.  $\frac{2}{3}$

**Question 14**

Find the sum of the real solutions of  $4^x - 3 \cdot 2^{x+1} + 8 = 0$ .

A.3

B.2

C.1

D. $\log_2 3$

E.6

F.8

G. $\log_2 6$

H.4

**Question 15**

Find the total area of the regions enclosed between the curve  $y = x^3 - x$  and the line  $y = 3x$ .

A.4

B. $\frac{9}{2}$ 

C.6

D. $\frac{15}{2}$ 

E.8

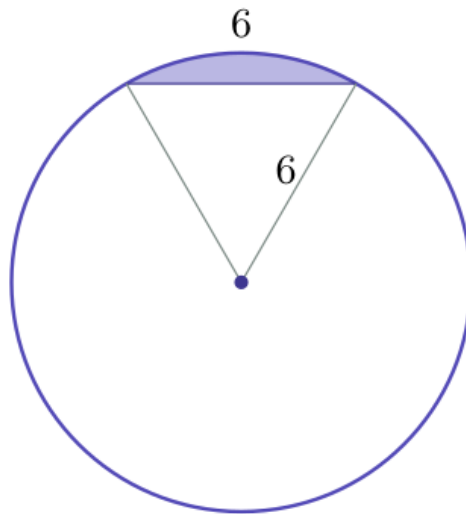
F.12

G.16

H. $\frac{32}{3}$

**Question 16**

A chord of length 6 is drawn in a circle of radius 6. The minor segment cut off is shaded.



What is the area of the shaded segment?

- A.  $6\pi - 18$
- B.  $6\pi - 9$
- C.  $12\pi - 9\sqrt{3}$
- D.  $9\sqrt{3} - 6\pi$
- E.  $6\pi - 18\sqrt{3}$
- F.  $12\pi - 18\sqrt{3}$
- G.  $3\pi - 9\sqrt{3}$
- H.  $6\pi - 9\sqrt{3}$

**Question 17**

How many ordered pairs  $(m, n)$  of positive integers satisfy  $m^2 - n^2 = 2025$  ?

A.3

B.4

C.5

D.6

E.7

F.8

G.12

H.15

**Question 18**

An angle  $\theta$  satisfies  $\sin \theta + \sin^2 \theta = 1$ . Find  $\cos^2 \theta + \cos^4 \theta$ .

A.1

B.0

C. $\frac{1}{2}$

D.2

E. $\frac{3}{2}$

F. $\sqrt{5} - 2$

G. $\frac{3-\sqrt{5}}{2}$

H.it depends on  $\theta$

**Question 19**

Find the complete set of values of  $k$  for which the equation  $x|x| - 2x + k = 0$  has three distinct real roots.

A.  $k > 0$

B.  $0 < k < 1$

C.  $-1 < k < 0$

D.  $-1 < k < 1$

E.  $0 \leq k \leq 1$

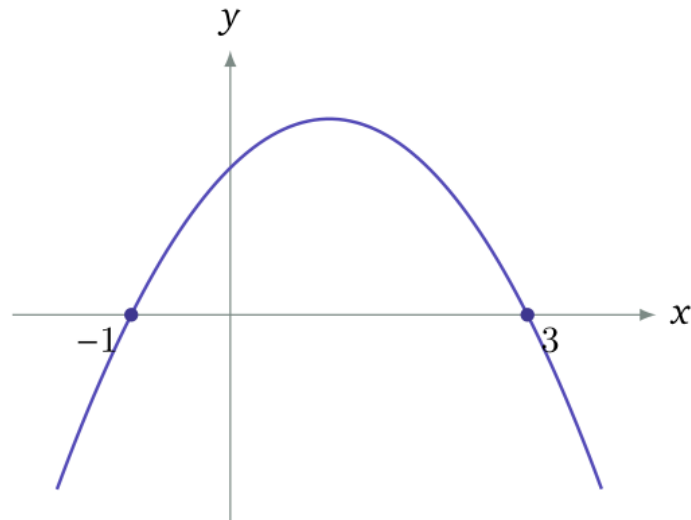
F.  $k < 1$

G.  $-1 \leq k \leq 1$

H. there is no such  $k$

**Question 20**

The diagram shows the graph of  $y = ax^2 + bx + c$ .



Which of the following is true?

A.  $a > 0, b > 0, c > 0$

B.  $a > 0, b > 0, c < 0$

C.  $a > 0, b < 0, c > 0$

D.  $a > 0, b < 0, c < 0$

E.  $a < 0, b < 0, c > 0$

F.  $a < 0, b < 0, c < 0$

G.  $a < 0, b > 0, c > 0$

H.  $a < 0, b > 0, c < 0$

## Answer key

1. Answer: F
2. Answer: C
3. Answer: G
4. Answer: B
5. Answer: A
6. Answer: B
7. Answer: F
8. Answer: D
9. Answer: C
10. Answer: B
11. Answer: H
12. Answer: D
13. Answer: C
14. Answer: A
15. Answer: E
16. Answer: H
17. Answer: E
18. Answer: A
19. Answer: D
20. Answer: G

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