TMUA Challenge Paper 2 Tmua-p2-challenge-1

20 questions. Recreated from the publicly available paper.

Question 1

For a real number x, consider the two statements $x > 2$ and $x^2 > 4$. The condition $x > 2$ is:

A.necessary and sufficient for $x^2 > 4$

B.necessary but not sufficient for $x^2 > 4$

C.sufficient but not necessary for $x^2 > 4$

D.neither necessary nor sufficient for $x^2 > 4$

E.equivalent to $x < -2$

Question 2

Consider the statement ”Every child in the class owns a bicycle.” Its negation is:

A.No child in the class owns a bicycle

B.Every child in the class does not own a bicycle

C.At least one child in the class does not own a bicycle

D.At least one child in the class owns a bicycle

E.Not every child in the class owns a car

Question 3

Which statement is logically equivalent to ”If it is raining then the ground is wet”?

A.If the ground is wet then it is raining

B.If the ground is not wet then it is not raining

C.If it is not raining then the ground is not wet

D.The ground is wet only if it is not raining

E.It is raining and the ground is dry

Question 4

A proof that there is no largest prime begins: ”Suppose, for contradiction, that p is the largest prime.” It then considers the number $N = (2 \times 3 \times 5 \times \cdots \times p) + 1$, the product of all primes up to p, plus one. The property of N that drives the contradiction is that:

A. N is itself prime

B. N is even

C. N leaves remainder 1 on division by every prime up to p

D. N is a perfect square

E. N is divisible by p

Question 5

It is known that any multiple of 12 is a multiple of 4. A particular integer n is not a multiple of 4. Which of the following can be deduced?

A. n is a multiple of 12

B. n is odd

C. n is not a multiple of 12

D. n is a multiple of 3

E. nothing can be deduced

Question 6

For how many of the first five prime numbers ($p = 2, 3, 5, 7, 11$) is the number $2^p - 1$ also prime?

A.2

B.3

C.4

D.5

E.1

Question 7

Consider the argument: ”All squares are rectangles. Some rectangles are not rhombuses. Therefore some squares are not rhombuses.” This argument is:

A.valid

B.invalid

C.valid only if at least one square exists

D.valid because every square is a rhombus

E.equivalent to its own converse

Question 8

For a positive integer n, the statement "n is divisible by 6" is logically equivalent to:

A.n is divisible by 2 or by 3

B.n is divisible by 2 and by 3

C.n is divisible by 12

D.n is even

E.the digits of n sum to a multiple of 6

Question 9

How many of the following statements are true for all real numbers x?

I. $x^2 \geq x$.

II. If $x > 1$ then $x^2 > x$.

III. If $x^2 > x$ then $x > 1$.

IV. $x^2 = x$ has exactly two real solutions.

A.1

B.2

C.3

D.4

E.0

Question 10

The statement ”For every real number x there is a real number y with $y^2 = x$” is false. Its correct negation is:

A.For every real x there is no real y with $y^2 = x$

B.There is a real x such that, for every real y, $y^2 \neq x$

C.There is a real y with $y^2 \neq x$ for every real x

D.For every real y there is a real x with $y^2 = x$

E.No real number has a real square root

Question 11

The implication ”$P \Rightarrow Q$” is false in exactly one of the following situations. Which one?

A.P true and Q true

B.P true and Q false

C.P false and Q true

D.P false and Q false

E.whenever P is false

Question 12

Consider this ”proof” that $2 = 1$. Let $a = b$. Then $a^2 = ab$, so $a^2 - b^2 = ab - b^2$, giving $(a + b)(a - b) = b(a - b)$, hence $a + b = b$; with $a = b$ this gives $2b = b$, so $2 = 1$. The first invalid step is:

A.writing $a^2 = ab$

B.writing $a^2 - b^2 = ab - b^2$

C.factorising both sides

D.cancelling the factor $(a - b)$

E.concluding $2b = b$

Question 13

Which of the following is a correct necessary and sufficient condition for a parallelogram to be a rectangle?

A.its diagonals are perpendicular

B.its diagonals are equal in length

C.all four sides are equal

D.it has a pair of parallel sides

E.its diagonals bisect each other

Question 14

Let A and B be sets with $A \subseteq B$. Which of the following must be true?

A.$B \subseteq A$

B.$A \cap B = B$

C.$A \cup B = A$

D.$A \cap B = A$

E.A and B are disjoint

Question 15

Consider the statement: ”For all integers a, b, c: if a divides bc then a divides b or a divides c.” This statement is:

A.true

B.false, and $a = 4$, $b = 2$, $c = 2$ is a counterexample

C.false, and $a = 5$, $b = 2$, $c = 3$ is a counterexample

D.false, and $a = 3$, $b = 2$, $c = 6$ is a counterexample

E.true, because it holds whenever a is prime

Question 16

For which real values of x is the statement "$x^2 - x + 1 > 0$" true?

A.only for $x > 0$

B.for all real x

C.for all $x \neq \frac{1}{2}$

D.for no real x

E.only for $x > 1$

Question 17

The converse of the statement ”If a function is differentiable at a point then it is continuous at that point” is:

A.If a function is continuous at a point then it is differentiable there

B.If a function is not differentiable at a point then it is not continuous there

C.If a function is not continuous at a point then it is not differentiable there

D.A function is differentiable at a point if and only if it is continuous there

E.If a function is continuous at a point then it is not differentiable there

Question 18

Three boxes are labelled ”Apples”, ”Oranges” and ”Mixed”. You are told that every label is wrong. You may draw a single fruit from one box of your choice and look at it. From which box should you draw in order to guarantee that you can then deduce the contents of all three boxes?

A.the box labelled ”Apples”

B.the box labelled ”Oranges”

C.the box labelled ”Mixed”

D.any single box will work

E.it cannot be done with a single draw

Question 19

The arithmetic-geometric mean inequality states that $u+v \geq 2\sqrt{uv}$ for all positive reals u, v. Using only this fact, the minimum value of $x + \frac{9}{x}$ for $x > 0$ is:

A.3

B.6

C.9

D.0

E.there is no minimum

Question 20

Let n be an integer and consider the statement "if n is odd then $n^2 + n$ is even". Which of the following are true?

I. The statement is true.

II. In fact $n^2 + n$ is even for every integer n.

III. The converse of the statement is true.

A.I only

B.I and II only

C.I, II and III

D.II only

E.III only

Answer key

1. Answer: C
2. Answer: C
3. Answer: B
4. Answer: C
5. Answer: C
6. Answer: C
7. Answer: B
8. Answer: B
9. Answer: B
10. Answer: B
11. Answer: B
12. Answer: D
13. Answer: B
14. Answer: D
15. Answer: B
16. Answer: B
17. Answer: A
18. Answer: C
19. Answer: B
20. Answer: B