JZ Mock Set A Paper 2

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

Consider the equation $x^3 - 3px^2 + 4 = 0$, where p is a real parameter.

Which of the following is a **sufficient** but **not necessary** condition on p for this equation to have exactly one real root?

A $p > 1$

B $p > -1$

C $0 < p < 2$

D $-2 < p < 2$

E $|p| < 1$

F $p^2 < 2$

Question 2

What is the gradient of the curve

$$y = \frac{(\sqrt{x}+2)^3}{x\sqrt{x}}$$

at the point where $x = 4$?

A $-\frac{5}{2}$

B $-\frac{3}{2}$

C $-\frac{7}{2}$

D $\frac{7}{2}$

E $\frac{5}{2}$

F $\frac{3}{2}$

Question 3

A student attempts to solve the inequality

$$\sqrt{x-3} < x - 2.$$

Consider the following attempt:

$$\text{Can only square-root non-negative, therefore } x - 3 \geq 0 \quad \Leftrightarrow \quad x \geq 3. \qquad \text{(I)}$$

$$\text{Squaring both sides gives } x - 3 < (x-2)^2. \qquad \text{(II)}$$

$$\text{Simplifies to } 0 < x^2 - 5x + 7. \qquad \text{(III)}$$

$$\text{The discriminant of } x^2 - 5x + 7 \text{ is } 25 - 28 = -3 < 0. \qquad \text{(IV)}$$

$$\text{Since the leading coefficient is positive, } x^2 - 5x + 7 > 0 \text{ for all real } x \geq 3. \qquad \text{(V)}$$

$$\text{Therefore the solution is } x \geq 3. \qquad \text{(VI)}$$

A The attempt is invalid, and the first problematic step is (I).

B The attempt is invalid, and the first problematic step is (II).

C The attempt is invalid, and the first problematic step is (III).

D The attempt is invalid, and the first problematic step is (IV).

E The attempt is invalid, and the first problematic step is (V).

F The attempt is invalid, and the first problematic step is (VI).

G The attempt is valid, and the solution is correct.

Question 4

Consider the following statement:

If a positive integer N has the property that N^2 is divisible by 12, then N is divisible by 12.

Which of the following are counterexamples to this statement?

I $N = 6$

II $N = 8$

III $N = 18$

IV $N = 24$

A none of them

B I only

C II only

D I and III only

E I, II and III only

F II and IV only

G I, III and IV only

H I, II, III and IV

Question 5

A student attempts to prove the following **claim:**

For every positive integer n, if $n^2 + 2$ is prime, then n is a multiple of 3.

The student's argument is given line by line below.

I. Suppose n is a positive integer such that $n^2 + 2$ is prime, but n is not a multiple of 3.

II. Then the remainder of n on division by 3 is 1 or 2.

III. In either case, n^2 leaves remainder 1 when divided by 3.

IV. Therefore $n^2 + 2$ leaves remainder 0 when divided by 3.

V. So 3 divides $n^2 + 2$.

VI. Hence $n^2 + 2 = 3k$ for some integer k.

VII. It follows that $n^2 + 2$ is composite and not prime.

VIII. This contradicts the assumption in (I) that $n^2 + 2$ is prime.

IX. Therefore n is a multiple of 3.

Which one of the following statements about the argument is correct?

A The argument is completely correct.

B The first error in the argument is on line I.

C The first error in the argument is on line II.

D The first error in the argument is on line III.

E The first error in the argument is on line IV.

F The first error in the argument is on line V.

G The first error in the argument is on line VI.

H The first error in the argument is on line VII.

I The first error in the argument is on line VIII.

J The first error in the argument is on line IX.

Question 6

Of the 200 books in a school library: 140 are hardback and 60 are paperback. 70 are illustrated and 130 are not illustrated. 170 are written in English and 30 are not written in English.

What is the smallest number of books that could be hardback, not illustrated, and written in English?

A 10

B 20

C 30

D 40

E 50

F 60

G 70

Question 7

The non-zero real numbers a, b, c and d satisfy $a < b$ and $c < d$.

Which of the following statements are **necessarily** true?

I $\frac{1}{a^3} > \frac{1}{b^3}$.

II $3^{-a} < 3^{-b}$.

III If a, b are negative, then $ac < bd$.

IV If a, b, c, d are all negative, then $ac > bd$.

A none of them

B **II** only

C **III** only

D **II** and **IV** only

E **I**, **II** and **III** only

F **II**, **III** and **IV** only

G **I**, **II** and **IV** only

H **IV** only

I **I** only

J **III** and **IV** only

Question 8

A sequence is defined by $a_1 = 1$ and

$$a_{n+1} = \frac{a_n}{1 + a_n}$$

for $n \geq 1$. Find the value of

$$\sum_{n=1}^{\infty} a_n a_{n+1}.$$

A 0

B 1

C 2

D 4

E $\frac{5}{2}$

F $\frac{3}{2}$

G ∞

Question 9

Find the complete set of values of k for which the graph $y = |2x + k|$ lies strictly below the curve $y = x^2 + (k-1)x + 5$ for every real value of x.

A $1 - 2\sqrt{3} < k < 1 + 2\sqrt{3}$

B $k < 1 + 2\sqrt{7}$

C $1 - 2\sqrt{5} < k < 1 + 2\sqrt{5}$

D $1 - 2\sqrt{3} < k < 1 + 2\sqrt{5}$

E $k < 1 - 2\sqrt{3}$ or $k > 1 + 2\sqrt{3}$

F There are no such values of k.

G $k < 1 - 2\sqrt{5}$ or $k > 1 + 2\sqrt{5}$

Question 10

The region R in the (x, y)-plane consists of all points satisfying **both**

$$|y - x^2| < 2 \qquad \text{and} \qquad x + y < 4.$$

Which of the following claims about points in R are true?

I: For every $(x, y) \in R$, $y \leq 6$.

II: For every $(x, y) \in R$, $y > -2$.

III: For every $(x, y) \in R$, $x \leq 2$.

A None of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 11

P: k is an integer multiple of π.

Q: $\int_0^k \left(\cos x + \cos^3(2x)\right)\, dx = 0$.

Which of the following best describes the logical relationship between P and Q?

A P is necessary and sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is neither necessary nor sufficient for Q.

Question 12

Consider the curve

$$y = \log_{x-1} 4 \quad \text{for } x > 1,\ x \neq 2.$$

Which of the following statements about this curve are true?

I $y > 0$ for every x in the domain.

II y is a strictly decreasing function of x on the domain.

III For every real number y, there exists a value of x in the domain such that $y = \log_{x-1} 4$.

IV $y = 0$ is an asymptote.

A I only

B II only

C III only

D IV only

E I and II only

F I and IV only

G II and III only

H III and IV only

I None of the statements is true.

J All four statements are true.

Question 13

Find the maximum value of

$$f(x) = \frac{2\sin^2 x + 10\cos x - 14}{\cos^2 x + 3\cos x - 10}$$

where x is a real number.

A $\frac{2}{3}$

B 1

C $\frac{5}{3}$

D 2

E $\frac{8}{3}$

F f has no maximum value

Question 14

Given curve $y = -\frac{1}{3}x^3 + a^2x + a$, has a local minimum in the second quadrant, find the possible values of a.

A $0 < a < \sqrt{\dfrac{3}{2}}$

B Any a such that $a \neq 0$

C $-\sqrt{\dfrac{3}{2}} < a < \sqrt{\dfrac{3}{2}}$ with $a \neq 0$

D $-\sqrt{3} < a < 0$

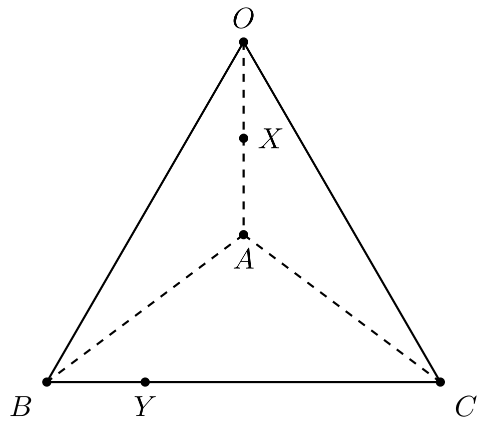
E $0 < a < \sqrt{3}$

F $-\sqrt{3} < a < \sqrt{3}$ with $a \neq 0$

G There are no possible values of a.

Question 15

A regular tetrahedron $OABC$ has every edge of length 12 metres. The point X is the midpoint of edge OA, and the point Y lies on edge BC with $BY = 3$ metres, as shown below.



What is the shortest distance, in metres, from X to Y measured entirely along the outer surface of the tetrahedron?

A $3\sqrt{3}$

B $3\sqrt{7}$

C $3\sqrt{13}$

D $3\sqrt{21}$

E 21

Question 16

A safe has three levers, A, B and C, each of which can be positioned either left or right at any particular time. The state of the safe (open or closed) depends only on the positions of these three levers. It is known that:

If lever A is right and (lever B is left or lever C is right), **then** the safe is open.

Which one of the following statements **must** be true?

A If the safe is open, then lever A is right and either lever B is left or lever C is right.

B If the safe is closed, then lever A is left, and either lever B is right or lever C is left.

C If the safe is closed, then lever A is left, lever B is right and lever C is left.

D If the safe is closed, then either lever A is left, or both lever B is right and lever C is left.

E If lever A is left, or both lever B is right and lever C is left, then the safe is closed.

F If the safe is open, then either lever A is left, or both lever B is right and lever C is left.

Question 17

In this question, n is an integer greater than 1.

Statement: If n does not divide $k!$ for any positive integer k with $k < n$, then n is prime.

The following is an attempted proof of the statement:

(1) Assume n does not divide $k!$ for any positive integer k with $k < n$.

(2) Suppose for contradiction that n is composite, so $n = ab$ for some integers a, b with $1 < a \leq b < n$.

(3) Case 1: $a < b$. Then a and b are distinct integers, each in the set $\{2, 3, \ldots, n-1\}$.

(4) Therefore a and b both appear as factors in the product $(n-1)! = 1 \cdot 2 \cdots (n-1)$, so $ab = n$ divides $(n-1)!$.

(5) Case 2: $a = b$, so $n = a^2$.

(6) Since $a \geq 2$, the integers a and $2a$ both satisfy $a \geq 2$ and $2a \geq 4$, so they are positive integers.

(7) Since $a \geq 2$, we have $a < a^2 = n$ and $2a < a^2 = n$, so a and $2a$ are distinct integers in the set $\{2, 3, \ldots, n-1\}$.

(8) Therefore a and $2a$ both appear as factors in $(n-1)!$, so $a \cdot 2a = 2a^2 = 2n$ divides $(n-1)!$, and in particular n divides $(n-1)!$.

(9) In both cases n divides $(n-1)!$, contradicting the assumption in line 1. Hence n is prime.

A First mistake appears in line 1 and the statement is false.

B First mistake appears in line 2 and the statement is false.

C First mistake appears in line 3 and the statement is false.

D First mistake appears in line 4 and the statement is false.

E First mistake appears in line 5 and the statement is false.

F First mistake appears in line 6 and the statement is false.

G First mistake appears in line 7 and the statement is false.

H The proof is valid and the statement is true.

Question 18

Let $a, b, c > 0$ with $a \neq 1$, $b \neq 1$ and $c \neq 1$. Consider the three equations

$$\log_a b = c, \qquad \log_b c = a, \qquad \log_c a = b.$$

Which one of the following statements about the solutions (a, b, c) of this system is correct?

A The equations specify a, b and c uniquely.

B The equations specify the product abc uniquely but have infinitely many solutions for (a, b, c).

C The equations specify exactly one of a, b, c uniquely but have infinitely many solutions for the other two.

D The equations have no solutions.

Question 19

Let $\lceil x \rceil$ denote the smallest integer that is greater than or equal to x. Compute

$$\int_0^{20} \lceil x \rceil \cdot 2^{\lceil x \rceil} \, dx.$$

A $\quad 20 \cdot 2^{21}$

B $\quad 19 \cdot 2^{21}$

C $\quad 18 \cdot 2^{20} + 2$

D $\quad 19 \cdot 2^{21} + 2$

E $\quad 19 \cdot 2^{20} + 2$

F $\quad 20 \cdot 2^{20} + 2$

Question 20

Given that p is an integer, find the minimum value of p such that the equation

$$\sqrt{\frac{1+\sin x}{1-\sin 2x}} = \sqrt{\frac{1+\sin 2x}{1-\sin x}}$$

has exactly 16 solutions in the interval $0^\circ \leq x < p^\circ$.

A 540

B 541

C 720

D 721

E 900

F 901

G 1080

H 1081