JZ Mock Set B Paper 2

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

It is a fact that

$$1729 = 1^3 + 12^3 = 9^3 + 10^3 = 7 \times 13 \times 19.$$

Consider the following four statements.

(1) Every positive integer that can be written as a sum of two positive cubes in two different ways is prime.

(2) If a prime $p > 5$ can be written as $p = a^3 + b^3$ for some positive integers a, b, then p has remainder of 1 when divided by 6.

(3) No product of three distinct odd primes is one more than a perfect cube.

(4) Every positive integer of the form $a^3 + b^3$, where a and b are positive integers, has prime factors all of which are at least 3.

The fact above can be used to provide a counterexample to which of these statements?

A none of them

B 1 only

C 3 only

D 1 and 3 only

E 2 and 3 only

F 1, 2 and 3 only

G 1, 3 and 4 only

H 1, 2, 3 and 4

Question 2

A student is asked to solve the equation

$$\frac{\sqrt{x^3+x^2-x}+2}{x+2}=1.$$

The student writes:

Step (1): Multiply both sides by $x+2$ to obtain

$$\sqrt{x^3+x^2-x}+2=x+2.$$

Step (2): Subtract both sides by 2 to get

$$\sqrt{x^3+x^2-x}=x.$$

Step (3): Square both sides to obtain

$$x^3+x^2-x=x^2.$$

Step (4): Rearrange to obtain

$$x^3-x=0.$$

Step (5): Factorise to obtain

$$x(x-1)(x+1)=0.$$

Step (6): Deduce

$$x=-1,0,1.$$

Which one of the following statements is correct?

A The final answers for x are wrong, and the first error appears in Step (1).

B The final answers for x are wrong, and the first error appears in Step (2).

C The final answers for x are wrong, and the first error appears in Step (3).

D The final answers for x are wrong, and the first error appears in Step (4).

E The final answers for x are wrong, and the first error appears in Step (5).

F The final answers for x are wrong, and the first error appears in Step (6).

G The final answers for x are correct, and there is no error in the student's working.

Question 3

In a TMUA question, students are asked to determine which, if any, of the four statements 1, 2, 3 and 4 are true. The options given in the question are:

A: no statements are true

B: only statement 1 is true

C: only statement 2 is true

D: only statements 1 and 2 are true

E: only statement 3 is true

F: only statements 1 and 3 are true

G: only statement 4 is true

H: only statements 2 and 3 are true

I: only statements 2 and 4 are true

Assume that a **competent student** can always correctly determine the truth value of any statement they choose to check.

If the student is lucky, what is the **smallest number of checks** that might let them determine the correct option? Equivalently, what is the fewest checks after which it is **possible** for the student to know the correct option?

A 1

B 2

C 3

D 4

Question 4

Find the exact value of

$$\tan 1^\circ \, \tan 2^\circ \, \tan 3^\circ \, \cdots \, \tan 89^\circ.$$

You may find the identity $\sin(x) = \cos(90^\circ - x)$ useful.

Remark: This is a well-known question in mathematical circles, but it is still valuable for students to encounter.

A 1

B 0

C $\sqrt{3}$

D 89

E $\frac{1}{2}$

Question 5

Let k be a real constant. Consider the polynomial functions

$$p(x) = 2kx^4 - x^2 + 5k \quad \text{and} \quad q(x) = kx^4 + x^2 + k.$$

Which of the following is the **necessary and sufficient** condition on k for $p(x) \geq q(x)$ for all real x?

A $k \geq 0$

B $k > 0$

C $k \geq \frac{1}{4}$

D $k \geq \frac{1}{2}$

E $k > \frac{1}{2}$

F $k \leq -\frac{1}{2}$

G $k \geq \frac{1}{2}$ or $k \leq -\frac{1}{2}$

H $-\frac{1}{2} \leq k \leq \frac{1}{2}$

Question 6

Given k is a real valued non-zero constant, and x is a non-zero real number, then

$$x^2 = \frac{k}{|x-2|}$$

has n distinct solutions. Which of the following is the **complete** list of possible values of n?

A $0,\ 2,\ 3,\ 4$

B $0,\ 1,\ 3,\ 4$

C $1,\ 2,\ 3,\ 4$

D $0,\ 1,\ 2,\ 3$

E $0,\ 2,\ 3$

F $1,\ 2,\ 3$

G $0,\ 1$

H $1,\ 2$

I $0,\ 3,\ 4$

Question 7

The circle C has equation

$$x^2 + y^2 - 6x + 4y - 12 = 0.$$

The point $P = (11, 2)$ lies outside C. The two tangents from P to C touch the circle at A and B.

Let Γ be the circle passing through P, A and B. What is the radius of Γ?

A $\sqrt{17}$

B $\sqrt{20}$

C $\sqrt{21}$

D $\sqrt{28}$

E $\sqrt{34}$

Question 8

The following is an attempted proof of the conjecture:

if $\sin\theta > \cos\theta$, **then** $\sin\theta > \frac{1}{\sqrt{2}}$.

Suppose $\sin\theta > \cos\theta$.

Then $\sin\theta - \cos\theta > 0$. (I)

Squaring the positive quantity $\sin\theta - \cos\theta$ gives $(\sin\theta - \cos\theta)^2 > 0$, which expands using $\sin^2\theta + \cos^2\theta = 1$ to $1 - 2\sin\theta\cos\theta > 0$, so $\sin\theta\cos\theta < \frac{1}{2}$. (II)

From (II), $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta < 2$, hence $|\sin\theta + \cos\theta| < \sqrt{2}$, so in particular $\sin\theta + \cos\theta > -\sqrt{2}$. (III)

Adding the inequalities (I) $\sin\theta - \cos\theta > 0$ and (III) $\sin\theta + \cos\theta > -\sqrt{2}$ gives $2\sin\theta > -\sqrt{2}$, so $\sin\theta > -\frac{1}{\sqrt{2}}$; squaring this inequality yields $\sin^2\theta > \frac{1}{2}$. (IV)

Since $\sin^2\theta > \frac{1}{2}$ means $|\sin\theta| > \frac{1}{\sqrt{2}}$, and $\sin\theta > -\frac{1}{\sqrt{2}}$ rules out $\sin\theta < -\frac{1}{\sqrt{2}}$, we conclude $\sin\theta > \frac{1}{\sqrt{2}}$. (V)

Which one of the following is the case?

A The proof is correct.

B The proof is incorrect, and the first error occurs in line (I).

C The proof is incorrect, and the first error occurs in line (II).

D The proof is incorrect, and the first error occurs in line (III).

E The proof is incorrect, and the first error occurs in line (IV).

F The proof is incorrect, and the first error occurs in line (V).

Question 9

Let k be a real constant. Consider the simultaneous equations

$$k \cdot 2^x + \log_2 y + \log_2 z = 3,$$

$$2^x + k \log_2 y + \log_2 z = 2,$$

$$2^x + \log_2 y + \log_2 z = k,$$

in real unknowns x, y and z, subject to $y > 1$ and $z > 0$.

Which of the following is the **necessary and sufficient** condition on k for the system to have a real-valued solution set $\{x, y, z\}$?

A $k > 1$

B $1 < k < 2$

C $1 < k < 3$

D $2 < k < 3$

E $k < 2$ and $k \neq 1$

F $k > 2$

G $k < 1$ or $k > 2$

H All real k except $k = 1$

Question 10

The equation

$$\sin x\,(1-\cos^2 x) = k\sin x$$

has exactly n distinct solutions in $[0, 2\pi]$, where k is a real constant.

Which of the following statements are true?

I $\quad 0 < k < 1$ **if** $n = 7$.

II $\quad n = 3$ **only if** $k < 0$ or $k > 1$.

III $\quad n = 5$ and $k \neq 1$ cannot both be true.

A Statement I is the only true statement.

B Statement II is the only true statement.

C Statement III is the only true statement.

D Statements I and III are the only true statements.

E Statements I and II are the only true statements.

F Statements II and III are the only true statements.

G All three statements are true.

H None of them are true.

Question 11

Let n be an integer. Which of the following is a **necessary but not sufficient** condition for which $n^3 + 5n$ is divisible by 12?

A n is 1 more than a multiple of 4.

B n is odd.

C n is even.

D n is a multiple of 3.

E n is a multiple of 4.

F n is a multiple of 6.

G n is 2 more than a multiple of 4.

Question 12

Let P be a property of a real number x. Each of the following is a statement about all real values of x. **Exactly one** of the following statements is true. Which one is it?

A If $|x| < 1$, then P is true.

B If $|x| < 2$, then P is true.

C If P is true, then $|x| < 1$.

D If P is true, then $|x| < 2$.

E If P is not true, then $x^2 \geq 1$.

F If P is not true, then $x^2 \geq 4$.

G P is true only if $x^2 < 1$.

Question 13

Positive real numbers x, y, z, none equal to 1, satisfy

$$x^{\log_y z} = y^{\log_z x} = z^{\log_x y} = 8.$$

What is the value of $(\log_2 x)(\log_2 y)(\log_2 z)$?

A 27

B 18

C 8

D 16

E 32

F not possible to determine

Question 14

For each positive integer k, let $F(k)$ be the sum of all real solutions x of

$$\sin(2^k x) = \frac{1}{2}$$

in the interval $0 \le x \le \frac{2\pi}{2^k}$. Find

$$\sum_{k=1}^{\infty} F(k).$$

A $\frac{\pi}{2}$

B $\frac{3\pi}{2}$

C π

D $\frac{3\pi}{4}$

E $\frac{5\pi}{2}$

F 2π

G The series diverges.

Question 15

A set X of positive integers, with $|X| \geq 3$, is called **cross-linked** if and only if for every number x in X there exist two distinct numbers y and z in X, both different from x, and a prime p, such that p divides each of x, y and z.

Let Y be a set of positive integers with $|Y| \geq 3$.

Which of the following statements is equivalent to Y being **not** cross-linked?

A For every number x in Y, and for every two distinct numbers y and z in Y with $y \neq x$ and $z \neq x$, there is no prime which divides all of x, y and z.

B For every number x in Y there exist two distinct numbers y and z in Y, both different from x, such that no prime divides all of x, y and z.

C There exists a number x in Y, and there exist two distinct numbers y and z in Y with $y \neq x$ and $z \neq x$, such that no prime divides all of x, y and z.

D There exists a number x in Y such that for every two distinct numbers y and z in Y with $y \neq x$ and $z \neq x$, there is no prime which divides all of x, y and z.

E There exists a number x in Y such that for every two distinct numbers y and z in Y with $y \neq x$ and $z \neq x$, there is a prime which divides x but divides neither y nor z.

F For every number x in Y there exist two distinct numbers y and z in Y, both different from x, and a prime p, such that p divides x but p does not divide y and p does not divide z.

Question 16

Let

$$W = \int_0^1 4^{\sqrt{x}}\,dx, \qquad X = \int_1^2 2^{\sqrt{x-1}}\,dx, \qquad Y = \int_0^{1/2} 2\cdot 2^{2x}\,dx, \qquad Z = \int_{-1}^1 \frac{1}{2}(\sqrt{2})^{(x+1)/2}\,dx.$$

Which of the following gives W, X, Y, Z in order from **smallest to largest**?

A $Z < Y < X < W$

B $W < X < Y < Z$

C $Z < X < Y < W$

D $Y < Z < X < W$

E $Z < Y < W < X$

F $X < Z < Y < W$

Question 17

Find the units digit of

$$7^{7^{7^{7^{7}}}}.$$

You may find it useful to consider the remainder of odd powers of 7 when divided by 4.

A 1

B 2

C 3

D 4

E 5

F 6

G 0

Question 18

Let f and g be polynomials. Consider the following four statements.

I: If $f'(x) \geq g'(x)$ for all $x \geq 0$ and $f(0) \geq g(0)$, then $f(x) \geq g(x)$ for all $x \geq 0$.

II: If $f(x) \geq g(x)$ for all $x \geq 0$, then $\int_0^x t\, f(t)\, \mathrm{d}t \geq \int_0^x t\, g(t)\, \mathrm{d}t$ for all $x \geq 0$.

III: If $\int_0^x f(t)\, \mathrm{d}t \geq \int_0^x g(t)\, \mathrm{d}t$ for all $x \geq 0$, then $f(x) \geq g(x)$ for all $x \geq 0$.

IV: If $\int_0^x f(t)\, \mathrm{d}t \geq \int_0^x g(t)\, \mathrm{d}t$ for all $x \geq 0$, then $\int_0^x t^2\, f(t)\, \mathrm{d}t \geq \int_0^x t^2\, g(t)\, \mathrm{d}t$ for all $x \geq 0$.

Which of the statements are true?

A None of them

B I only

C II only

D III only

E IV only

F I and II only

G II and III only

H I and III only

I all except I

J all except II

K all except III

L all except IV

Question 19

A circle has centre $A = (6, 7)$. Two tangents from the point $B = (1, 2)$ touch the circle at P and Q. Given that length $BP = 2\sqrt{10}$, find the sum of the gradients of the tangent lines BP and BQ.

A $\frac{6}{3}$

B $\frac{3}{4}$

C $\frac{8}{3}$

D $\frac{3}{2}$

E $\frac{10}{3}$

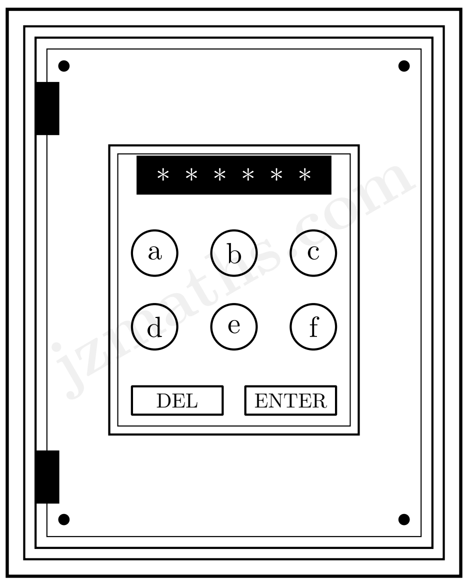
F $\frac{7}{4}$

G $\frac{5}{4}$

Question 20

Disclaimer: This is a **very hard** version of a similar question from the past exam questions! Expect to give yourself an extra 10-15 minutes to solve this. You will learn a great deal from trying!

A safe is protected by a 6-character password that uses each of the letters a, b, c, d, e, f exactly once. After each attempt the safe reports only the number of positions in which the attempted string agrees with the password (it does not say which positions). The following three attempts have already been made: $abcdef$ gave 0 correct positions, $feadbc$ gave 0 correct positions, and $fbdcae$ gave 3 correct positions. A player now plays optimally, choosing each subsequent attempt with full knowledge of all responses received so far. What is the smallest number N such that the player is guaranteed to be able to identify the password after at most N further attempts (regardless of what the password actually is)?



A 1

B 2

C 3

D 4

E more than 5