JZ Mock Set C Paper 1

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

Find

$$\int_{-1}^{1} 1 + 2x + 3x^2 + 4x^3 + ... + 11x^{10}\, dx.$$

A 11

B 10

C 12

D 22

E 0

F 2

Question 2

Find the number of real solutions of $|x^2 - 1| = 2^x$.

A 1

B 2

C 3

D 4

E 5

F 6

Question 3

The real numbers a, b, c satisfy

$$4 - |a-8| \geq 0, \qquad |2b-10| \leq 6, \qquad |c-8| \leq 7.$$

What is the least possible value of $|abc|$?

A 15

B 12

C 10

D 32

E 8

F 0

Question 4

Which of the following values is the **largest**?

A $\dfrac{7}{5}$

B $\dfrac{17}{12}$

C $\dfrac{10}{7}$

D $\dfrac{11}{8}$

E $2\cos\left(\dfrac{\pi}{4}\right)$

Question 5

How many distinct real solutions does $4^x + 6^x = 9^x$ have?

A 0

B 1

C 2

D 3

E 4

Question 6

Three distinct numbers are chosen at random from $\{1, 2, \ldots, 10\}$. Find the probability that, arranged in increasing order, they form an arithmetic progression.

A $\dfrac{1}{4}$

B $\dfrac{1}{5}$

C $\dfrac{1}{6}$

D $\dfrac{1}{8}$

E $\dfrac{2}{9}$

F $\dfrac{2}{15}$

Question 7

A line l with positive gradient is a common tangent to the circles $x^2 + y^2 = 4$ and $(x-9)^2 + y^2 = 25$, such that the two circles lie on the same side of l.

Find the gradient of l.

A $\frac{1}{3}$

B $2\sqrt{2}$

C $\frac{\sqrt{2}}{4}$

D $\frac{3\sqrt{2}}{8}$

E $\frac{\sqrt{2}}{2}$

F $\frac{2\sqrt{2}}{3}$

G $\frac{3\sqrt{2}}{4}$

H $\frac{7\sqrt{2}}{8}$

Question 8

For positive integers n let:

$$I(n) = \int_0^{2n} \frac{|n-x|}{n}\, dx$$

What is the largest positive integer n for which

$$\sum_{r=1}^{n} I(r) < 210?$$

A 15

B 16

C 19

D 20

E 21

Question 9

Three players A, B and C take turns throwing a fair six-sided die in the order A, B, C, A, B, C, ... until one of them wins. The win conditions on a player's own throw are:

$$A \text{ wins if } A \text{ throws a } 6,$$
$$B \text{ wins if } B \text{ throws a } 5 \text{ or a } 6,$$
$$C \text{ wins if } C \text{ throws a } 4, 5 \text{ or } 6.$$

If a player throws and does not win, play passes to the next player. What is the probability that B is the first to win?

A $\dfrac{3}{13}$

B $\dfrac{5}{18}$

C $\dfrac{5}{13}$

D $\dfrac{6}{13}$

E $\dfrac{1}{3}$

F $\dfrac{5}{9}$

Question 10

The curve $x = (y-2)^2 - 1$ is transformed as follows:

First, it is translated vertically in the positive y-direction by 2 units.

Next, it is reflected in the line $x = 2$.

Finally, it is rotated 90° clockwise about the point $(1, 1)$.

Which of the following is the equation of the resulting curve?

A $y = x^2 - 8x + 13$

B $y = x^2 - 8x + 19$

C $y = -x^2 + 8x - 13$

D $x = y^2 - 8y + 15$

E $y = x^2 - 4x + 1$

Question 11

Find the sum of all distinct solutions to

$$\sin(5x + 90^\circ) = \cos 4x$$

for $0 \leq x \leq 360^\circ$, where x is measured in degrees.

A 900

B 1800

C 1440

D 1080

E 2160

Question 12

What is the smallest positive value of k for which the line $x = k$ is a line of symmetry of **both** of the curves

$$y = \sin\left(2x - \frac{\pi}{3}\right) \quad \text{and} \quad y = \cos\left(3x + \frac{\pi}{4}\right)?$$

A $\frac{\pi}{12}$

B $\frac{\pi}{4}$

C $\frac{5\pi}{12}$

D $\frac{7\pi}{12}$

E $\frac{11\pi}{12}$

F $\frac{23\pi}{12}$

Question 13

Find the units digit of $2^{2026} + 3^{2026}$.

A 1

B 3

C 5

D 7

E 9

Question 14

The function f is defined for all real x by

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}kx^3 - k^2x^2 + k,$$

where k is a real constant. Given that the equation $f(x) = 0$ has four distinct real roots, what are the possible values of k?

A $\quad k > \sqrt[3]{\frac{12}{5}}$

B $\quad k < -\sqrt[3]{\frac{12}{5}} \quad \text{or} \quad k > \sqrt[3]{\frac{12}{5}}$

C $\quad k > \sqrt[3]{\frac{3}{8}}$

D $\quad k < 0 \quad \text{or} \quad k > \sqrt[3]{\frac{12}{5}}$

E $\quad 0 < k < \sqrt[3]{\frac{12}{5}}$

F $\quad k > \sqrt{\frac{12}{5}}$

Question 15

A sequence is defined by $a_0 = \frac{1}{2}$, and $a_n = 2\sum_{r=0}^{n-1} a_r$ for $n \geq 1$. Evaluate

$$\sum_{r=1}^{\infty} \frac{1}{a_r + a_{r+1}}.$$

A the sum does not converge

B $\dfrac{3}{8}$

C $\dfrac{5}{6}$

D $\dfrac{4}{5}$

E $\dfrac{5}{9}$

Question 16

The set of points satisfying

$$\tan(x^2 + y^2) = 1, \qquad x^2 + y^2 < k\pi,$$

is made up of finitely many separate closed curves. For each separate closed curve, consider the region enclosed by that curve.

Given that the sum of the areas of these regions is $30\pi^2$, find the smallest possible integer value of k.

A 5

B 6

C 7

D 8

E 9

F 10

Question 17

A rectangle has its sides parallel to the coordinate axes and lies entirely within the region enclosed by $y = 2|x - 4|$ and $y = x + 6$.

What is the maximum possible area of the rectangle?

You may assume that the rectangle with maximum possible area has its bottom edge meeting both arms of the graph $y = 2|x - 4|$.

A $\frac{40}{3}$

B 16

C 16.5

D $\frac{49}{3}$

E 15

F $\frac{50}{3}$

G $\frac{35}{2}$

Question 18

A positive integer k is chosen uniformly at random from $1, 2, 3, \ldots, 8$.

For each non-negative integer n, define

$$a_n = (2k+1) \sum_{r=0}^{n} \binom{n}{r} \left(\frac{k-4}{4} \right)^r .$$

Let

$$S = a_0 + a_1 + a_2 + \cdots .$$

What is the probability that S converges to a finite value and $S > 8$?

A $\frac{1}{8}$

B $\frac{1}{4}$

C $\frac{3}{8}$

D $\frac{1}{2}$

E $\frac{5}{8}$

F $\frac{3}{4}$

G $\frac{7}{8}$

H 1

I 0

Question 19

A triangle ABC has $AB = x$, $BC = 2$, and $\angle BAC = 30°$. Find the range of values of x for which there are two non-congruent possible triangles satisfying these conditions, and in both triangles $\angle ABC$ is acute.

A $0 < x < 2$

B $2 < x < 4$

C $2 < x < 2\sqrt{3}$

D $2\sqrt{3} < x < 4$

E $x > 2\sqrt{3}$

F $0 < x < 2\sqrt{3}$

Question 20

The function f is defined by

$$f(x) = \ln\left(\frac{1+x}{1-x}\right), \qquad -1 < x < 1.$$

Solve

$$f(x) = 4f\left(\frac{1}{\sqrt{2}}\right).$$

It may be helpful to first find constants a and b such that, for $u, v \in (-1, 1)$,

$$f\left(\frac{u+v}{1+uv}\right) = af(u) + bf(v).$$

A $\frac{3\sqrt{2}}{4}$

B $\frac{8\sqrt{2}}{9}$

C $\frac{4\sqrt{2}}{5}$

D $\frac{12\sqrt{2}}{17}$

E $\frac{7\sqrt{2}}{10}$