JZ Maths

JZ Mock Set C Paper 2

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Spotted an error? Please email jzmaths@hotmail.com.

Question 1

Let n denote a positive integer. Consider the following three statements.

I. For every n, if n^2 is divisible by 12, then n is divisible by 6.

II. For every n, if n^2 is divisible by 12, then n is divisible by 12.

III. There exists an n such that n^2 is divisible by 12 but n is not divisible by 12.

Which of the statements are true?

A I only

B II only

C III only

D I and II only

E II and III only

F I and III only

G all of them

Question 2

Let x, y, z be real numbers with $x \geq y$. Which of the following statements **must** be true?

1. $x^3 \geq y^3$
2. $x^2 \geq y^2$
3. $xz \geq yz$
4. $xz^2 \geq yz^2$

A none of them

B 1 only

C 4 only

D 1 and 2 only

E 1 and 3 only

F 1 and 4 only

G 2 and 4 only

H 1, 2 and 4 only

I 1, 3 and 4 only

J 1, 2, 3 and 4

Question 3

Given that $\sin\theta + \cos\theta = \frac{1}{2}$, find the value of $\sin^3\theta + \cos^3\theta$.

A $\dfrac{1}{4}$

B $\dfrac{5}{8}$

C $\dfrac{7}{8}$

D $\dfrac{7}{12}$

E $\dfrac{11}{16}$

F $\dfrac{13}{18}$

Question 4

Consider the following statements about the polynomial $f(x)$, where $a < b$:

(1) $f(a) > f(b)$;

(2) $f'(x) \leq 0$ for all $x \in [a, b]$;

(3) $\int_a^b f(x)\, dx \leq 0$;

(4) $f(a) + f(b) \leq 2f\left(\frac{a+b}{2}\right)$.

Which of these statements is a **necessary** condition for $f(x)$ to be decreasing for $a \leq x \leq b$? Here, decreasing means that $x_1 \leq x_2$ implies $f(x_1) \geq f(x_2)$.

A none of them

B Only 1.

C Only 2.

D Only 3.

E Only 4.

F Only 1 and 2.

G Only 1 and 3.

H Only 1 and 4.

I Only 2 and 3.

J Only 2 and 4.

Question 5

Let $y(x)$ be a polynomial defined for all real values of x.

Consider the following statements:

Statement P: The function y has a local maximum at $x = 0$.

Statement Q: $y'(0) = 0$ and $y''(0) < -1$.

Which one of the following is true?

A P is neither necessary nor sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is necessary and sufficient for Q.

E Whether P is necessary or sufficient for Q depends on the degree of y.

Question 6

Let $x \neq 0$ and

$$f(x) = \frac{3}{7}x^{7/3} + \frac{3}{4}x^{4/3} - 6x^{1/3}.$$

A student attempts to find the values of x for which the curve is increasing.

Their working is shown below.

I. $f'(x) = x^{4/3} + x^{1/3} - 2x^{-2/3}$

II. $f'(x) = x^{-2/3}(x^2 + x - 2)$

III. $x^{-2/3}(x^2 + x - 2) \geq 0$

IV. $x^2 + x - 2 \geq 0$

V. $(x - 1)(x + 2) \geq 0$

VI. $x \leq -2$ or $x \geq 1$

Which of the following is true?

A The first error is in step **I**.

B The first error is in step **II**.

C The first error is in step **III**.

D The first error is in step **IV**.

E The first error is in step **V**.

F The first error is in step **VI**.

G The student's answer is correct.

Question 7

Six sealed black bags are labelled P, Q, R, S, T and U. Each bag contains the same number of balls. This number is a non-zero whole number from 3 to 8 inclusive.

Each bag has a label with a statement written on it. The statement may be true or false.

Bag P has a label which says: "The number of balls in this bag is a multiple of 3."

Bag Q has a label which says: "The number of balls in this bag is even."

Bag R has a label which says: "The number of balls in this bag is prime."

Bag S has a label which says: "The number of balls in this bag is at most 4."

Bag T has a label which says: "The number of balls in this bag is at least 7."

Bag U has a label which says: "The number of balls in this bag is a perfect square."

Given that exactly one of the six statements is true, which bag has the true statement on its label?

A P

B Q

C R

D S

E T

F U

Question 8

A polynomial $p(x)$ satisfies $p(2) = 7$ and $p(-3) = -8$.

Which of the following can be deduced, where $q(x)$ denotes some polynomial?

A $p(x) = (x+2)(x-3)\,q(x) + 3x + 1$

B $p(x) = (x-2)(x+3)\,q(x) - 3x + 1$

C $p(x) = (x-2)(x+3)\,q(x) + 3x - 1$

D $p(x) = (x-2)(x+3)\,q(x) + 3x + 1$

E $p(x) = (x-2)(x+3)\,q(x) - x + 5$

F $p(x) = (x-2)(x+3)\,q(x) + x - 5$

Question 9

How many ordered pairs of integers (x, y) satisfy

$$(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)=63?$$

A 0

B 1

C 2

D 3

E 4

F 6

G 8

H infinitely many

Question 10

In a TMUA-style question, students are asked to determine which, if any, of the five statements $1, 2, 3, 4, 5$ are true. They must then identify the correct option from the list provided in the question.

Assume that a **competent student** can always correctly determine the truth value of any statement they choose to check.

A teacher is designing such a question and wants to choose the option set that **maximises the smallest number of checks** a lucky student might need to determine the correct option. Equivalently, the teacher wants to maximise the fewest checks after which it is **possible** for the student to know the correct option.

Which of the following four candidate option sets best achieves the teacher's aim?

Option Set I

A: only statement 1 is true
B: only statements $1, 2, 3$ are true
C: only statements $1, 2, 4$ are true
D: only statements $1, 4, 5$ are true
E: only statements $1, 5$ are true

Option Set II

A: only statement 1 is true
B: only statement 2 is true
C: only statement 3 is true
D: only statement 4 is true
E: only statement 5 is true

Option Set III

A: only statements $1, 2$ are true
B: only statements $2, 3$ are true
C: only statements $3, 4$ are true
D: only statements $4, 5$ are true
E: no statements are true

Option Set IV

A: only statements $1, 2$ are true
B: only statements $2, 3$ are true
C: only statements $3, 4$ are true
D: only statements $4, 5$ are true
E: only statements $1, 5$ are true

A **Option Set I**

B **Option Set II**

C **Option Set III**

D **Option Set IV**

Question 11

In triangle ABC, $\angle A = 30^\circ$, $AB = 2x^2$ and $BC = 3x$, where $x > 0$. Suppose that there are exactly two non-congruent triangles satisfying these conditions. Which of the following conditions can be **deduced**?

A $2 < x < \frac{7}{2}$

B $\frac{5}{3} < x < 5$

C $1 < x < 4$

D $\frac{1}{2} < x < \frac{5}{2}$

E $0 < x < 2$

Question 12

A student makes the following **claim**:

If f is a real-valued function such that, for some constant a, $\dfrac{f(x)+f(-x)}{2} = a$ for all real values of x, then there exists a constant k such that $f(x) = kx + a$ for all real values of x.

Examine their **claim** above, and determine which of the following is true?

A The **claim** is true.

B $f(x) = 2x + 1$ is a counterexample to the claim.

C $f(x) = \sin x$ is a counterexample to the claim.

D $f(x) = \tan x$ is a counterexample to the claim.

E $f(x) = \cos x$ is a counterexample to the claim.

F $f(x) = -x$ is a counterexample to the claim.

Question 13

Let x and y be non-zero real numbers with $x \neq y$.

Which one of the following statements is both **necessary and sufficient** for $x < y$?

A $y^{-1/3} < x^{-1/3}$

B $y^{1/5} < x^{1/5}$

C $x^{2/3} < y^{2/3}$

D $y^{2/3} < x^{2/3}$

E $|x|^{1/3} < |y|^{1/3}$

F $x^{-1/3} < y^{-1/3}$

G $x^{1/5} < y^{1/5}$

Question 14

For which of the following is it a **necessary but not sufficient** condition that

$$2x^3 - 3px^2 + p = 0$$

has exactly one real root?

A $-1 < p < 1$

B $-1 \leq p \leq 1$

C $0 < p < 1$

D $p = 0$

E $p \leq 0$

F $p \geq 0$

Question 15

Which one of the following numbers is largest in value?

A $\log_2 7$

B $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

C $\dfrac{1}{\sqrt{3}-\sqrt{2}}$

D $\sqrt{12}$

E $\sqrt{6+4\sqrt{2}}$

Question 16

Three transformations of the plane are defined as follows. R is the reflection in the y-axis. T is the translation by 4 units in the positive x-direction. S is the horizontal stretch with scale factor 2 parallel to the x-axis, fixing the y-axis.

Let $y = f(x)$ be a function defined on the real numbers.

When the graph of $y = f(x)$ is transformed by applying R, then T, then S in that order, the result is the graph of $y = g(x)$.

When the graph of the same function $y = f(x)$ is transformed by applying S, then T, then R in that order, the result is the graph of $y = h(x)$.

Which one of the following conditions on $y = f(x)$ is **necessary and sufficient** for the functions $g(x)$ and $h(x)$ to be identical?

A $f(x) = f(x + 3)$ for all x

B $f(x) = f(x + 4)$ for all x

C $f(x) = f(x + 6)$ for all x

D $f(x) = f(x + 12)$ for all x

E $f(x) = f(-x)$ for all x

F $f(x) = f(6 - x)$ for all x

Question 17

Determine the total number of ordered pairs of positive integers (x, y) satisfying

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{9}.$$

A 5

B 6

C 8

D 9

E 10

F 12

Question 18

Let $f(x) = x^3 - px^2 - p^2x + k$ where k is a real constant. Suppose that p and q are real numbers satisfying $-3 \leq p \leq 3$ and $-3 \leq q \leq 3$.

The function f has non-negative gradient at $x = q$.

Find the area of the region of all possible points (p, q) in the p-q plane.

A 20

B 21

C 24

D 30

E 36

F 42

G 56

H 60

Question 19

An infinite sequence of integers $u_1, u_2, u_3, \dots$ satisfies $u_1 = 10$ and

$$u_n + 2 < u_{n-1} < u_n + 9 \quad \text{for } n > 1.$$

It is given that $u_k = -50$ and that there exists some integer j with $1 < j < k$ such that $u_j = -10$.

How many different values can k take?

Disclaimer: This question is inspired by a question from tylertutoring.com. I found the original question very interesting and useful, so I created a new version with a couple of additional twists. Perhaps a few too many?! Enjoy!

A 4

B 9

C 10

D 12

E 13

F 15

G 36

H 48

Question 20

A quadrilateral is convex if each of its interior angles is less than $180°$.

Let $ABCD$ be a convex quadrilateral, with its vertices labelled in order. A quadrilateral is called a rhombus if all four of its sides are equal in length.

Which of the following statements, taken individually, are **sufficient** to guarantee that $ABCD$ is a rhombus?

1. $AB = AD$, and the diagonals AC and BD intersect at right angles.

2. $AB = CD$, and the diagonals AC and BD divide $ABCD$ into four similar triangles.

3. The diagonals AC and BD divide $ABCD$ into four similar triangles.

4. The diagonal AC divides $ABCD$ into two triangles, and the diagonal BD divides $ABCD$ into two triangles, with all four of these triangles having the same area.

5. $AB = BC = CD$, and the triangles ABC and BCD are congruent.

A Only 1.

B Only 2.

C Only 4.

D Only 5.

E Only 1 and 3.

F Only 2 and 3.

G Only 2 and 4.

H Only 3 and 4.

I Only 5 and 1.

J Only 2 and 5.

K Only 1 and 2.

L None of them.