JZ Mock Set D Paper 1

Recommended time: 2 hours

Calculators: not permitted

Format: 20 multiple-choice questions

This is a TMUA-style mock paper modelled on the Test of Mathematics for University Admission. The TMUA is used in admissions for mathematics, economics, computer science, and engineering courses at universities including Cambridge, Oxford, Imperial College London, UCL, LSE, Warwick, and Durham.

Paper Guidance

- These papers contain only hard, very hard, and occasionally *unreasonably hard* TMUA-style questions :). I have deliberately left out the routine and easier material, so please do not be discouraged if you find a paper difficult or need longer than the recommended time. That is exactly how it is meant to feel. Take your time, enjoy the challenge, and remember that struggling with difficult problems is one of the best ways to get better.
- As a rough and deliberately conservative guide, a student who scores above 27 out of 40 across one full set, meaning both papers in four hours, is very likely already working at around the 7.0+ level on the current TMUA scale. However, this is only intended as a guide, not something to worry about too much. The main purpose of these papers is improvement rather than measurement: meet difficult problems, struggle with them, learn from them, and focus on getting better each time.

Best of luck, and enjoy the challenge!

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Question 1

Find the coefficient of x^4 in the expansion of

$$\left(2 - x + \frac{x^2}{2}\right)^5.$$

A 50

B 60

C 70

D 80

E 90

Question 2

How many solutions are there to the equation

$$|\tan x| + \frac{1}{\tan x} = 3 \quad \text{for} \quad 0^\circ \leq x \leq 360^\circ?$$

A 2

B 4

C 5

D 6

E 8

Question 3

The functions f and g are defined for all real x by

$$f(x) = |x - 2| + |x + 2|$$

and

$$g(x) = x^2 - 2x + 4.$$

The function h is defined by

$$h(x) = g(f(x)).$$

What is the minimum value of $h(x)$?

A 10

B 11

C 12

D 13

E 14

F 15

Question 4

Find the total length of interval(s) on which

$$f(x) = \frac{2}{5}x^{5/2} - 2x^{3/2} + 4x^{1/2} \qquad x > 0$$

is decreasing.

A $\frac{1}{2}$

B 1

C 2

D 3

E 4

F $\frac{5}{2}$

Question 5

Find the area of the region in the (x, y)-plane consisting of all points satisfying both

$$|x| + |y| \leq 2 \quad \text{and} \quad |x - 1| + |y| \leq 2.$$

A 3

B $\frac{9}{4}$

C $\frac{7}{2}$

D 4

E $\frac{9}{2}$

F 6

G $\frac{15}{2}$

H 8

Question 6

An arithmetic progression has $(2n + 1)$ terms, where n is a positive integer. Let S_o denote the sum of the terms in odd-numbered positions (the 1st, 3rd, 5th, ... terms) and let S_e denote the sum of the terms in even-numbered positions (the 2nd, 4th, 6th, ... terms).

It is given that

$$S_o + S_e = 505 \qquad \text{and} \qquad S_o - S_e = 5.$$

What is the value of n?

A 40

B 50

C 51

D 55

E 56

Question 7

Find the value of

$$\sum_{r=1}^{100} \frac{1}{\sqrt{r}+\sqrt{r-1}}.$$

A 0

B 1

C 9

D 10

E 11

F 20

Question 8

The two curves C_1 and C_2 in the xy-plane have equations

$$x^2 + y^2 - 2y - 12 = 0 \quad \text{and} \quad x^2 + y^2 - 2x - 4y = 0.$$

Let P be a point on C_1 and Q be a point on C_2. What is the **least** possible value of the length of PQ?

A $\sqrt{2} + \sqrt{5} - \sqrt{13}$

B $\sqrt{13} - \sqrt{5} - \sqrt{2}$

C $\sqrt{13} - \sqrt{2}$

D 0

E $\sqrt{13} + \sqrt{5} - \sqrt{2}$

Question 9

Let x be any real number, find the minimum value of

$$4^{\sin^2 x} - 5 \cdot 2^{\sin^2 x} + 7.$$

A There is no minimum value.

B 1

C $\frac{5}{4}$

D 3

E 7

F $\frac{3}{4}$

Question 10

Given that $x^2 - 4x + 1$ is a factor of

$$2x^4 - 11x^3 + bx^2 + cx + 6,$$

what is the value of $b + c$?

A -47

B -27

C -7

D 13

E 47

F 53

Question 11

Let $f(x) = -x^3 + kx^2 + 3$, where k is a real constant. Suppose that $f(x) = 0$ has exactly 3 distinct real roots.

Which of the following equations must also have exactly 3 distinct real roots?

I $-f(x) + a = 0$, where a is an arbitrary real constant

II $5 - 2f(2 - 3x) = 0$

III $f(-|x|) - 3 = 0$

A I only

B II only

C III only

D I and III only

E I and II only

F all of them

G II and III only

H none of them

Question 12

The curve $x = y^2 - 8y + 19$ is rotated 90° clockwise about $P_1 = (1, 2)$, and then 90° clockwise about P_2. The final curve is $x = -y^2 + 2y - 7$. Given P_2 lies on the line $y = 5$, find the x-coordinate of P_2.

A -1

B -2

C -3

D 0

E 1

F 2

G 3

H 4

Question 13

The graph of $y = \sin x$ is transformed by the following five operations, applied in the order given:

I a reflection in the line $y = -3$;

II a stretch by scale factor 2 parallel to the x-axis;

III a translation of $\frac{\pi}{3}$ in the negative x-direction.

IV a reflection in the line $x = 0$;

V a reflection in the line $x = \frac{4\pi}{3}$.

Which of the following is the equation of the resulting curve?

A

$$y = -3 + \sin\left(\frac{x}{2} + \frac{5\pi}{12}\right)$$

B

$$y = -6 - \sin\left(\frac{x}{2} - \frac{5\pi}{12}\right)$$

C

$$y = -6 + \sin\left(\frac{x}{2} - \frac{\pi}{6}\right)$$

D

$$y = -3 + \sin\left(\frac{x}{2} - \frac{5\pi}{12}\right)$$

E

$$y = 6 + \sin\left(\frac{x}{2} - \frac{5\pi}{12}\right)$$

Question 14

The equation

$$\cos x + \cos^3 x + \cos^5 x + \cos^7 x + \ldots = \tan x$$

has how many solutions for $0 \leq x < 2\pi$?

A 0

B 1

C 2

D 3

E 4

F infinitely many

Question 15

The circle C is inscribed in the triangle bounded by the three lines

$$y = 0, \qquad y = \sqrt{3}x, \qquad x + y = \sqrt{1} + \sqrt{2} + \sqrt{3}.$$

You may find $\tan 22.5^\circ = \sqrt{2} - 1$ useful.

Which of the following points lies inside C?

A $\left(\frac{1}{2}, 1\right)$

B $(2, 2)$

C $\left(\frac{3}{2}, \frac{5}{2}\right)$

D $\left(1, \frac{5}{3}\right)$

E $\left(3, \frac{3}{4}\right)$

Question 16

Let f be a real-valued function such that

$$\int_0^1 f(x)\,dx = 1$$

and $f(x) = f(x+1)$ for all real x.

Find

$$\sum_{r=1}^{100} \int_0^r f(r^r x)\,dx.$$

A 50

B 100

C 101

D 5050

E 2050

F 4950

Question 17

A real number x satisfies

$$x^2 - 3x + 1 = 0.$$

For each non-negative integer n, define

$$p_n = x^n + \frac{1}{x^n}.$$

Find the value of p_6.

You may find it helpful to consider a recurrence relation of the form $p_n = 3p_{n-1} + kp_{n-2}$ for some constant k.

A 322

B 123

C 312

D 151

E 276

F 258

Question 18

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

Find the value of

$$\int_{100}^{1} \log_{10}\left(\frac{\lfloor 2x \rfloor}{\lfloor 2x \rfloor + 1}\right) dx.$$

A -1

B $\log_{10} 99$

C $\log_{10} 101$

D $-\log_{10} 99$

E $-\log_{10} 101$

F 1

Question 19

Let functions $f_0(x) = |x|$ and $f_{n+1}(x) = |1 - f_n(x)|$, find

$$\int_0^{100} f_{100}(x)\,dx.$$

A 5000

B 2500

C 100

D 50

E 25

Question 20

The positive real numbers a, b, c, d form a geometric progression in this order. Given that

$$a + d = 35 \qquad \text{and} \qquad b + c = 30,$$

find the value of

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}.$$

A $\dfrac{65}{216}$

B $\dfrac{13}{7}$

C $\dfrac{13}{6}$

D $\dfrac{1}{216}$

E $\dfrac{5}{216}$

F $\dfrac{13}{210}$