



Clavis TMUA Set A Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Clavis

Question 1

Find the value of

$$\int_1^9 \frac{(\sqrt{x} - 1)^2}{\sqrt{x}} dx$$

A $\frac{4}{3}$

B $\frac{8}{3}$

C $\frac{14}{3}$

D $\frac{16}{3}$

E 6

F $\frac{20}{3}$

Question 2

An arithmetic progression has first term 3 and common difference d , where $d \neq 0$.

The 1st, 4th and 13th terms of this arithmetic progression form a geometric progression.

Find the sum of the first 10 terms of the arithmetic progression.

- A** 30
- B** 75
- C** 99
- D** 120
- E** 130
- F** 165

Question 3

The circles C_1 and C_2 have equations

$$C_1 : x^2 + y^2 = 25 \quad C_2 : (x - a)^2 + (y - a)^2 = 9$$

where a is a real constant.

Find the complete set of values of a for which C_1 and C_2 intersect at two distinct points.

- A** $-4\sqrt{2} < a < 4\sqrt{2}$
- B** $-8 < a < -2$ or $2 < a < 8$
- C** $-4\sqrt{2} < a < -\sqrt{2}$ or $\sqrt{2} < a < 4\sqrt{2}$
- D** $-\sqrt{2} < a < \sqrt{2}$
- E** $\sqrt{2} < a < 4\sqrt{2}$
- F** $a < -4\sqrt{2}$ or $a > 4\sqrt{2}$

Question 4

How many solutions does the equation

$$\sin 2x = \tan x$$

have in the interval $0 \leq x \leq 2\pi$?

A 3

B 4

C 5

D 6

E 7

F 8

G 9

Question 5

The equation

$$9^x - k 3^x + 27 = 0$$

has two distinct real solutions for x , where k is a real constant.

Find the complete set of values of k .

- A** $k < -6\sqrt{3}$
- B** $-6\sqrt{3} < k < 6\sqrt{3}$
- C** $0 < k < 6\sqrt{3}$
- D** $k < -6\sqrt{3}$ or $k > 6\sqrt{3}$
- E** $k > 6\sqrt{3}$
- F** $k > 108$

Question 6

In the triangle ABC , $AB = 5$, $AC = 8$, and the area of the triangle is $10\sqrt{3}$.

Find the sum of the two possible values of BC^2 .

- A** 49
- B** 80
- C** 89
- D** 129
- E** 178
- F** 218

Question 7

In the expansion of $(2 + ax)^6$, where a is a non-zero constant, the coefficient of x^2 is equal to the coefficient of x^3 .

Find the value of a .

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D 1

E $\frac{4}{3}$

F $\frac{3}{2}$

Question 8

The curve with equation $y = x^3 + ax^2 + bx$ has a stationary point at $(2, -4)$.

Find the value of $a + b$.

A -9

B -6

C -3

D 0

E 3

F 6

Question 9

The curve $y = x^2 + kx + 10$ lies entirely above the line $y = 2x + 1$.

Find the complete set of possible values of k .

- A** $-8 < k < 4$
- B** $-4 < k < 8$
- C** $k < -4$ or $k > 8$
- D** $-4 \leq k \leq 8$
- E** $-2\sqrt{10} < k < 2\sqrt{10}$
- F** all real values of k

Question 10

Find the area of the finite region enclosed by the curve $y = x(4 - x)$ and the line $y = x$.

A $\frac{3}{2}$

B $\frac{5}{2}$

C $\frac{9}{2}$

D $\frac{27}{4}$

E 9

F $\frac{27}{2}$

G 18

Question 11

Evaluate

$$\sum_{n=1}^{80} \log_4(2^n)$$

- A** 810
- B** 1620
- C** 2430
- D** 3240
- E** 6480

Question 12

The graph of $y = f(x)$ has a maximum point at $(3, 5)$.

The graph of $y = 2f(x - 1) + 4$ has a maximum point at (p, q) .

Find the value of $p + q$.

A 11

B 14

C 16

D 18

E 20

F 22

Question 13

The points $A(1, 2)$ and $B(7, 10)$ are the endpoints of a diameter of a circle.

The point P lies on the circle and $PA = 8$.

Find the length of PB .

A 4

B $2\sqrt{7}$

C 6

D 8

E $6\sqrt{2}$

F 10

G 12

Question 14

The trapezium rule with 4 strips of equal width is used to estimate

$$\int_0^4 (x^2 + 1) \, dx$$

Find the positive difference between this estimate and the exact value of the integral.

A $\frac{2}{3}$

B 1

C $\frac{4}{3}$

D 2

E $\frac{8}{3}$

F $\frac{10}{3}$

Question 15

A convergent geometric series has first term a and common ratio r .

The sum to infinity of the series is 9, and the sum of the first two terms is 5.

Find the sum of the possible values of a .

- A** 3
- B** 12
- C** 15
- D** 18
- E** 21

Question 16

How many integers x satisfy

$$|x - 2| + |x + 3| \leq 9$$

- A** 8
- B** 9
- C** 10
- D** 11
- E** 12
- F** infinitely many

Question 17

A set of 12 numbers has mean 7.

The two numbers 4 and 10 are removed from the set and replaced by two numbers, each equal to k .

The mean of the new set of 12 numbers is 8.

Find the value of k .

A 10

B 11

C 12

D 13

E 14

Question 18

A bag contains 3 red counters and 2 blue counters.

Two counters are removed at random, one after the other, without replacement.

Given that the second counter removed is red, find the probability that the first counter removed is red.

A $\frac{3}{10}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{2}{3}$

F $\frac{3}{4}$

Question 19

The equation $x^3 - 12x + q = 0$, where q is a real constant, has a repeated root.

Find the complete set of values of q .

- A** $q = -16$ or $q = 16$
- B** $q = -16$ only
- C** $q = 16$ only
- D** $q = -8$ or $q = 8$
- E** $q = 0$ only
- F** there are no such values of q

Question 20

A sector of a circle has radius r and angle θ radians, where $\theta < 2$.

The perimeter of the sector is 20 and its area is 21.

Find the value of r .

A 2

B 3

C 4

D 5

E 6

F 7