TMUA

COMMUNITY PAPERS

Clavis TMUA Set A Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Clavis

Question 1

The function f is defined for $x > 0$ by

$$\mathrm{f}(x) = \frac{x^2 + 6\sqrt{x}}{x}$$

Find the value of $\mathrm{f}'(9)$.

A $\frac{1}{9}$

B $\frac{8}{9}$

C $\frac{26}{27}$

D 1

E $\frac{28}{27}$

F $\frac{10}{9}$

Question 2

Find the coefficient of x^3 in the expansion of

$$(1-2x)^5(1+x)^2$$

A -80

B -30

C -10

D 10

E 30

F 80

Question 3

Consider the following claim:

For all real numbers a and b, if $a^2 > b^2$ then $a > b$.

Which of the following is/are a counterexample to the claim?

I $a = 3$, $b = -4$

II $a = -3$, $b = 2$

III $a = -5$, $b = -2$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 4

The sequence $u_1, u_2, u_3, ...$ is defined by $u_n = n^2 + kn$, where k is a real constant.

Consider the two statements:

P: $k > -2$

Q: the sequence is strictly increasing, that is $u_{n+1} > u_n$ for every $n \geq 1$

Which one of the following is true?

A P is necessary and sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is not necessary and not sufficient for Q.

Question 5

Let n be a positive integer, and let P be the statement

if n is a multiple of 4, then n is even.

Which of the following must be true?

I P

II the converse of P

III the contrapositive of P

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 6

Consider the following statement $(*)$ about a real constant k:

$(*)$ There exists a real number x such that, for all real numbers y,

$$x^2 + y^2 \geq kxy$$

What is the complete set of values of k for which $(*)$ is true?

A $-2 \leq k \leq 2$

B $-1 \leq k \leq 1$

C $k \geq 0$

D $k = 0$ only

E all real values of k

F there are no such values of k

Question 7

Here is an attempted proof of the following claim.

For all real numbers x, if $x^3 > x$ then $x > 1$.

(I) Suppose $x^3 > x$.

(II) Dividing both sides by x gives $x^2 > 1$.

(III) Therefore $x^2 - 1 > 0$, so $(x-1)(x+1) > 0$.

(IV) Hence $x > 1$ or $x < -1$.

(V) Therefore $x > 1$.

Which of the following is the case?

A The proof is correct.

B The proof is incorrect, and the first error occurs in line (I).

C The proof is incorrect, and the first error occurs in line (II).

D The proof is incorrect, and the first error occurs in line (III).

E The proof is incorrect, and the first error occurs in line (IV).

F The proof is incorrect, and the first error occurs in line (V).

Question 8

In this question a and b are real numbers with $a < b$, and

$$f(x) = (x - a)^2(x - b)$$

Which of the following statements must be true?

I f has exactly two stationary points.

II $f(x) < 0$ for all $x < a$.

III $f(x) \geq 0$ for all $x > a$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 9

A positive integer is called *balanced* if and only if it can be written as the sum of two or more consecutive positive integers. For example, 9 is balanced since $9 = 4 + 5$, and 15 is balanced since $15 = 7 + 8$.

A (true) theorem states that:

A positive integer is balanced if and only if it is not a power of 2.

So $1, 2, 4, 8, 16, ...$ are precisely the positive integers that are not balanced.

Which one of the following statements is not true?

A Every odd number greater than 1 is balanced.

B If N is balanced, then $2N$ is balanced.

C If N and M are balanced, then NM is balanced.

D If N is not balanced, then $N + 1$ is balanced.

Question 10

The curve C has equation $y = ax^2 + bx + c$, where a, b and c are real constants with $a \neq 0$.

Let d be the x-coordinate of the vertex of C.

Which one of the following is sufficient on its own to determine the value of d ?

A the values of a and c

B the values of b and c

C the values of a and b

D the value of the discriminant $b^2 - 4ac$

E none of the options above is sufficient on its own

Question 11

Which of the following statements is/are true?

I For every real number x there is a real number y such that $y^2 = x$.

II There is a real number y such that for every real number x, $xy = 1$.

III For every real number x there is a real number y such that $x + y = 0$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 12

Consider the following statement:

For every positive integer n there is a positive integer m such that $m^2 + n$ is a perfect square.

Which one of the following is the negation of this statement?

A For every positive integer n there is a positive integer m such that $m^2 + n$ is not a perfect square.

B For every positive integer n and every positive integer m, $m^2 + n$ is not a perfect square.

C There is a positive integer n such that there is a positive integer m for which $m^2 + n$ is not a perfect square.

D There is a positive integer n such that for every positive integer m, $m^2 + n$ is not a perfect square.

E There is a positive integer n such that for every positive integer m, $m^2 + n$ is a perfect square.

F There is a positive integer m such that for every positive integer n, $m^2 + n$ is not a perfect square.

G For every positive integer m there is a positive integer n such that $m^2 + n$ is not a perfect square.

H There are positive integers n and m such that $m^2 + n$ is not a perfect square.

Question 13

Consider the following attempt to solve the equation $2\sin x\cos x = \cos x$ for $0 \leq x \leq 2\pi$.

(I) Dividing both sides by $\cos x$ gives $2\sin x = 1$.

(II) So $\sin x = \frac{1}{2}$.

(III) The solutions of $\sin x = \frac{1}{2}$ with $0 \leq x \leq 2\pi$ are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

(IV) Therefore the solutions of the original equation are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

Which one of the following is true?

A The working is correct.

B The two values found are not solutions of the original equation, and the error arises as a result of step (I).

C The two values found are solutions, but there is exactly one further solution, missed as a result of step (I).

D The two values found are solutions, but there are exactly two further solutions, missed as a result of step (I).

E The two values found are solutions, but there are exactly two further solutions, missed as a result of step (III).

F The two values found are solutions, but there are exactly four further solutions, missed as a result of step (I).

Question 14

n is a positive integer.

Consider the statement:

Q: n is a multiple of 6

and the following three conditions:

I n is a multiple of 2 and n is a multiple of 3

II n is a multiple of 12

III n is a multiple of 2 or n is a multiple of 3 (or both)

Which row in the following table is correct?

	Condition I is necessary and sufficient for Q	Condition II is necessary and sufficient for Q	Condition III is necessary and sufficient for Q
A	yes	yes	yes
B	yes	yes	no
C	yes	no	yes
D	yes	no	no
E	no	yes	yes
F	no	yes	no
G	no	no	yes
H	no	no	no

A A

B B

C C

D D

E E

F F

G G

H H

Question 15

In this question f and g are polynomials such that

$$\mathrm{f}(x) \leq \mathrm{g}(x) \quad \text{for all real } x$$

Which of the following statements must be true?

I $\quad \mathrm{f}(\mathrm{g}(x)) \leq \mathrm{g}(\mathrm{g}(x))$ for all real x

II $\quad \mathrm{f}'(x) \leq \mathrm{g}'(x)$ for all real x

III $\quad \int_0^1 \mathrm{f}(x)\,\mathrm{d}x \leq \int_0^1 \mathrm{g}(x)\,\mathrm{d}x$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 16

Consider the following statement $(*)$ about a real number x:

$(*)$ There exists a real number y such that $y^2 + xy + x = 0$.

What is the complete set of values of x for which $(*)$ is true?

A $x \leq 0$ or $x \geq 4$

B $0 \leq x \leq 4$

C $x < 0$ or $x > 4$

D $0 < x < 4$

E all real values of x

F there are no such values of x

Question 17

Place the following three numbers in order of size, starting with the smallest.

$$P = 2^{300} \qquad Q = 3^{200} \qquad R = 5^{150}$$

A $P < Q < R$

B $P < R < Q$

C $Q < P < R$

D $Q < R < P$

E $R < P < Q$

F $R < Q < P$

Question 18

The function f is a polynomial defined for all real x.

Which one of the following is a necessary condition for

$$\mathrm{f}(x) \geq \mathrm{f}(0) \quad \text{for all real } x$$

A $\mathrm{f}(x) \geq 0$ for all real x

B $\mathrm{f}'(0) = 0$

C $\mathrm{f}'(x) \geq 0$ for all real x

D $\mathrm{f}''(0) > 0$

E $\mathrm{f}''(x) \geq 0$ for all real x

F f is an even function

Question 19

A function f defined for all real x is called *self-inverse* if and only if

$$\mathrm{f}(\mathrm{f}(x)) = x \quad \text{for all real } x$$

Which of the following statements is/are true?

I For every constant c, the function $\mathrm{f}(x) = c - x$ is self-inverse.

II If f and g are self-inverse, then the function $\mathrm{h}(x) = \mathrm{f}(\mathrm{g}(x))$ is self-inverse.

III Every self-inverse function is of the form $\mathrm{f}(x) = c - x$ for some constant c.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 20

It is given that $f(x) = x^3 + px + q$, where p and q are real constants, and that the equation $f(x) = 0$ has three distinct real roots.

Which of the following statements is/are necessarily true?

I $p > 0$

II The graph of $y = f(x)$ has a local maximum point with a positive y-coordinate.

III The equation $f(x) = 0$ has a rational root.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III