TMUA

COMMUNITY PAPERS

Clavis TMUA Set B Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Clavis

Question 1

The function f is defined for $x > 0$ by

$$\mathrm{f}(x) = \frac{(2x-3)^2}{\sqrt{x}}$$

Find the value of $\mathrm{f}'(1)$.

A $-\frac{9}{2}$

B -4

C $-\frac{3}{2}$

D 0

E $\frac{3}{2}$

F $\frac{9}{2}$

Question 2

Find the value of

$$\int_0^2 (3x-1)^2 \, \mathrm{d}x$$

A $\frac{26}{3}$

B 10

C $\frac{38}{3}$

D 14

E $\frac{46}{3}$

F 18

Question 3

Consider the following claim:

For all real numbers x, if $x^2 > x$ then $x > 1$.

Which of the following is/are a counterexample to the claim?

I $x = -2$

II $x = \frac{1}{2}$

III $x = 2$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 4

Let n be a positive integer, and consider the two statements:

P: n^2 is a multiple of 9

Q: n is a multiple of 3

Which one of the following is true?

A P is necessary and sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is not necessary and not sufficient for Q.

Question 5

Let n be a positive integer, and let P be the statement

if n is a multiple of 10, then n is a multiple of 5 and n is even.

Which of the following must be true?

I P

II the converse of P

III the contrapositive of P

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 6

Consider the following statement $(*)$ about a real number k:

$(*)$ For all real numbers x, $x^2 + kx + k > 0$.

What is the complete set of values of k for which $(*)$ is true?

A there are no such values of k

B $0 < k < 4$

C $0 \leq k \leq 4$

D $k < 0$ or $k > 4$

E $k \leq 0$ or $k \geq 4$

F $k > 0$

G all real values of k

Question 7

Here is an attempted proof of the following claim.

If $\sin\theta = \cos\theta$ then $\theta = \frac{\pi}{4}$.

(I) Suppose $\sin\theta = \cos\theta$.

(II) Squaring both sides gives $\sin^2\theta = \cos^2\theta$.

(III) Since $\sin^2\theta + \cos^2\theta = 1$, it follows that $2\cos^2\theta = 1$, so $\cos^2\theta = \frac{1}{2}$.

(IV) Hence $\cos\theta = \frac{1}{\sqrt{2}}$.

(V) Therefore $\theta = \frac{\pi}{4}$.

Which of the following is the case?

A The proof is correct.

B The proof is incorrect, and the first error occurs in line (I).

C The proof is incorrect, and the first error occurs in line (II).

D The proof is incorrect, and the first error occurs in line (III).

E The proof is incorrect, and the first error occurs in line (IV).

F The proof is incorrect, and the first error occurs in line (V).

Question 8

The function f is defined for all real x by

$$\mathrm{f}(x) = x|x|$$

Which of the following statements must be true?

I f is an even function.

II f is strictly increasing.

III $\mathrm{f}(x) = 0$ only when $x = 0$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 9

A positive integer is called *tidy* if and only if all of its digits, written in base 10, are different from one another. For example 1023 and 7 are tidy, but 121 and 330 are not.

Which one of the following statements is not true?

A Every one-digit positive integer is tidy.

B The sum of two tidy numbers is always tidy.

C There are only finitely many tidy numbers.

D Every tidy number has at most 10 digits.

Question 10

The line L has equation $y = mx + c$, and d is the distance from the origin to L.

Which one of the following is sufficient on its own to determine the value of d ?

A the value of m

B the value of c

C the value of the x-intercept of L

D the values of m and c

E none of the options above is sufficient on its own

Question 11

Which of the following statements is/are true?

I For every positive integer n there is a positive integer m such that $m < n$.

II There is a real number x such that for every real number y, $x < y$.

III For every real number x there is a real number y such that $xy = 1$.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 12

Consider the following statement:

There is a positive integer n such that for every positive integer m, $m + n$ is composite.

Which one of the following is the negation of this statement?

A There is a positive integer n such that for every positive integer m, $m + n$ is prime.

B For every positive integer n there is a positive integer m such that $m + n$ is prime.

C For every positive integer n and every positive integer m, $m + n$ is prime.

D For every positive integer n there is a positive integer m such that $m + n$ is composite.

E There is a positive integer n and a positive integer m such that $m + n$ is prime.

F For every positive integer m there is a positive integer n such that $m + n$ is prime.

G There is a positive integer m such that for every positive integer n, $m + n$ is prime.

H For every positive integer n and every positive integer m, $m + n$ is composite.

Question 13

Consider the following attempt to solve the equation $(x-3)(x+1) = 2(x-3)$.

(I) Dividing both sides by $(x-3)$ gives $x+1 = 2$.

(II) Therefore $x = 1$.

(III) So the only solution is $x = 1$.

Which one of the following is true?

A The working is correct.

B The working is incorrect: $x = 1$ is not a solution of the original equation, and the error arises as a result of step (I).

C The working is incorrect: there is a further solution of the original equation, and the error arises as a result of step (I).

D The working is incorrect: there is a further solution of the original equation, and the error arises as a result of step (II).

E The working is incorrect: there is a further solution of the original equation, but every step shown is valid.

F The working is incorrect: the original equation has no solutions.

Question 14

In this question f is a function defined for all real x.

Consider the following three conditions:

I $\quad f(-x) = -f(x)$ for all real x

II $\quad f(0) = 0$

III $\quad f(|x|) = |f(x)|$ for all real x

Which of these conditions is/are necessary and sufficient for the graph of $y = f(x)$ to have rotational symmetry of order 2 about the origin?

Which row in the following table is correct?

	Condition I is necessary and sufficient	Condition II is necessary and sufficient	Condition III is necessary and sufficient
A	yes	yes	yes
B	yes	yes	no
C	yes	no	yes
D	yes	no	no
E	no	yes	yes
F	no	yes	no
G	no	no	yes
H	no	no	no

A A

B B

C C

D D

E E

F F

G G

H H

Question 15

The real numbers a and b satisfy $a < b$.

Which of the following statements must be true?

I $a^2 < b^2$

II $a^3 < b^3$

III $\frac{1}{a} > \frac{1}{b}$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 16

Consider the following statement $(*)$ about a real number x:

$(*)$ For every real number y, $x + y^2 \geq 2y$.

What is the complete set of values of x for which $(*)$ is true?

A $x < 1$

B $x \leq 1$

C $x \geq 1$

D $x > 1$

E all real values of x

F there are no such values of x

Question 17

The function f is a polynomial with $f(x) > 0$ for all real x, and the function g is defined for all real x by

$$g(x) = \int_0^x f(t)\,dt$$

Which of the following statements must be true?

I $g(x) > 0$ for all real x.

II g is strictly increasing.

III $g(-x) = -g(x)$ for all real x.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 18

The function f is a non-zero polynomial defined for all real x.

Which one of the following is a necessary condition for the equation $f(x) = 0$ to have at least two distinct real roots?

A f has degree at least 3

B $f(0) = 0$

C $f''(x) = 0$ has at least one real solution

D $f'(x) < 0$ for all real x

E $f'(x) = 0$ has at least one real solution

F f has even degree

Question 19

A positive integer is called *stepped* if and only if its digits, read from left to right, are strictly increasing. For example 137 and 5 are stepped, but 132 and 771 are not.

Which of the following statements is/are true?

I Every stepped number has at most 9 digits.

II No stepped number contains the digit 0.

III If n is stepped then $2n$ is stepped.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 20

The function f is defined for all real x and satisfies

$$\mathrm{f}(x+2) = \mathrm{f}(x) \quad \text{for all real } x$$

Which of the following statements is/are necessarily true?

I $\mathrm{f}(x+6) = \mathrm{f}(x)$ for all real x.

II $\mathrm{f}(1) = \mathrm{f}(2)$.

III If f is a polynomial then f is constant.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III