TMUA

COMMUNITY PAPERS

Clavis TMUA Set C Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Clavis

Question 1

Find the value of

$$\int_0^1 (2x-1)^3 \, \mathrm{d}x$$

A $-\frac{1}{4}$

B $-\frac{1}{8}$

C 0

D $\frac{1}{8}$

E $\frac{1}{4}$

F $\frac{1}{2}$

Question 2

Given that

$$y = \frac{x^2 + 1}{\sqrt{x}} \quad \text{for } x > 0$$

find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1$.

A -1

B 0

C $\frac{1}{2}$

D 1

E $\frac{3}{2}$

F 2

Question 3

The numbers a, b and c are real, and $ab = ac$.

Which of the following statements must be true?

I $a = 0$

II $b = c$

III $a = 0$ or $b = c$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 4

Let a and b be positive integers, and consider the two statements:

P: ab is divisible by 6

Q: a is divisible by 6 or b is divisible by 6

Which one of the following is true?

A P is necessary and sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is not necessary and not sufficient for Q.

Question 5

Which of the following statements is/are true?

I Every integer greater than 2 can be written as the sum of two prime numbers.

II Every integer greater than 11 can be written as the sum of two integers greater than 1, neither of which is prime.

III Every prime number greater than 3 is of the form $6n + 1$ for some positive integer n.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 6

Which one of the following is smallest in value?

A $\left(\frac{1}{3}+\frac{1}{6}\right)^{-1}$

B $\log_2 5$

C $\sqrt{6}$

D $\frac{5}{2}$

E $2^{3/2}$

F $\frac{15}{7}$

G $3-\frac{1}{2}$

Question 7

Here is an attempted proof of the following claim.

If a and b are real numbers with $a^2 = b^2$, then $a = b$.

(I) Suppose $a^2 = b^2$.

(II) Then $a^2 - b^2 = 0$.

(III) Factorising gives $(a - b)^2 = 0$.

(IV) Hence $a - b = 0$.

(V) Therefore $a = b$.

Which of the following is the case?

A The proof is correct.

B The proof is incorrect, and the first error occurs in line (I).

C The proof is incorrect, and the first error occurs in line (II).

D The proof is incorrect, and the first error occurs in line (III).

E The proof is incorrect, and the first error occurs in line (IV).

F The proof is incorrect, and the first error occurs in line (V).

Question 8

Eight chairs are arranged in a circle, and some of them are occupied.

A chair is called *crowded* if both of the chairs next to it are occupied. (A chair may be crowded whether or not it is itself occupied.)

Which of the following statements is/are true?

I It is possible for exactly 4 chairs to be occupied with no chair crowded.

II If 5 chairs are occupied then at least one chair must be crowded.

III If exactly 2 chairs are occupied then no chair is crowded.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 9

The function f is defined on the positive integers by $f(1) = 3$ and, for $n \geq 1$,

$f(n+1) = 3\,f(n) + 1$ if $f(n)$ is odd

$f(n+1) = \frac{1}{2}\,f(n)$ if $f(n)$ is even

Find the value of $f(8)$.

A 1

B 2

C 4

D 5

E 8

F 16

Question 10

The trapezium rule is used to estimate $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, where $a < b$.

Which one of the following conditions guarantees that the estimate is an overestimate?

A $\mathrm{f}(x) > 0$ for all x with $a \leq x \leq b$

B $\mathrm{f}'(x) > 0$ for all x with $a \leq x \leq b$

C $\mathrm{f}''(x) > 0$ for all x with $a \leq x \leq b$

D $\mathrm{f}''(x) < 0$ for all x with $a \leq x \leq b$

Question 11

Let x be a real number.

Which one of the following statements is a sufficient condition for exactly three of the other four statements?

A $x = 2$

B $x > 0$

C $x \geq 2$

D $x > 1$

E $x \neq 0$

Question 12

Consider the following statement:

For every real number x there is a real number y such that $xy > 1$.

Which one of the following is the negation of this statement?

A For every real number x there is a real number y such that $xy \leq 1$.

B For every real number x and every real number y, $xy \leq 1$.

C There is a real number x such that there is a real number y with $xy \leq 1$.

D There is a real number x such that for every real number y, $xy > 1$.

E There is a real number y such that for every real number x, $xy \leq 1$.

F For every real number y there is a real number x such that $xy \leq 1$.

G There is a real number x such that for every real number y, $xy \leq 1$.

H There are real numbers x and y such that $xy \leq 1$.

Question 13

Consider the following attempt to solve the equation $\log_2 x + \log_2(x-2) = 3$.

(I) Combining the logarithms gives $\log_2 (x(x-2)) = 3$.

(II) Hence $x(x-2) = 8$.

(III) So $x^2 - 2x - 8 = 0$, which factorises as $(x-4)(x+2) = 0$.

(IV) Therefore the solutions are $x = 4$ and $x = -2$.

Which one of the following is true?

A The working is correct.

B Only one of $x = 4$ and $x = -2$ is a solution of the original equation, and the error arises as a result of step (I).

C Only one of $x = 4$ and $x = -2$ is a solution of the original equation, and the error arises as a result of step (II).

D Only one of $x = 4$ and $x = -2$ is a solution of the original equation, and the error arises as a result of step (III).

E Neither $x = 4$ nor $x = -2$ is a solution of the original equation.

F There is a further solution of the original equation that this working has missed.

Question 14

In this question f is a polynomial.

Consider the following three conditions:

I $\int_0^a \mathrm{f}(x)\,\mathrm{d}x = 0$ for every real number a

II $\mathrm{f}(x) = 0$ for all real x

III $\mathrm{f}(-x) = -\mathrm{f}(x)$ for all real x

Which of these conditions is/are necessary and sufficient for

$$\int_p^q \mathrm{f}(x)\,\mathrm{d}x = 0 \quad \text{for all real numbers } p \text{ and } q\,?$$

Which row in the following table is correct?

	Condition I is necessary and sufficient	Condition II is necessary and sufficient	Condition III is necessary and sufficient
A	yes	yes	yes
B	yes	yes	no
C	yes	no	yes
D	yes	no	no
E	no	yes	yes
F	no	yes	no
G	no	no	yes
H	no	no	no

A A

B B

C C

D D

E E

F F

G G

H H

Question 15

Class X contains m students, and their mean mark in a test is 60.

Class Y contains n students, and their mean mark in the same test is 70.

The mean mark of all the students in the two classes combined is 64.

Which of the following statements must be true?

I $m > n$

II m is a multiple of 3

III If the mean mark of class Y had been 74 instead (with all class sizes and the class X marks unchanged), the combined mean mark would have been 66.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 16

A bag contains 5 red counters and 3 blue counters.

Three counters are drawn at random, without replacement.

Consider the following three probabilities:

P the probability that all three counters are red

Q the probability that exactly one counter is red

R the probability that none of the counters is red

Place P, Q and R in order of size, starting with the smallest.

A $P < Q < R$

B $P < R < Q$

C $Q < P < R$

D $Q < R < P$

E $R < P < Q$

F $R < Q < P$

Question 17

Let n be a positive integer.

Which one of the following is a necessary but not sufficient condition for n to be a perfect square?

A n is odd

B n has an odd number of positive divisors

C n is a multiple of 4

D $n + 1$ is a perfect square

E n is prime

F the last digit of n is not 2, 3, 7 or 8

Question 18

The real numbers a, b and c satisfy $a < b < c < 0$.

Which of the following statements must be true?

I $ab < ac$

II $a + b < b + c$

III $ac > bc$

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 19

For a positive integer n, let $\mathrm{d}(n)$ denote the number of positive divisors of n. For example $\mathrm{d}(6) = 4$, since 6 has divisors $1, 2, 3$ and 6.

Call n *compact* if and only if $\mathrm{d}(n) = 4$.

Which of the following statements is/are true?

I Every compact number is even.

II Every compact number is the product of two distinct primes.

III If n is compact then $2n$ is compact.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III

Question 20

In this question n is a positive integer and $N = n^4 + 4$.

Which of the following statements is/are true?

I N is composite for every integer $n > 1$.

II N is divisible by 5 for every positive integer n.

III N is a perfect square for some positive integer n.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III