TMUA

COMMUNITY PAPERS

DPiD Maths Mock Set 1 Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: DPiD

Question 1

What is the smallest prime divisor of $3^{2^{(5^{12})}} + 1$?

A 2

B 3

C 5

D 7

E 11

F 13

G 17

H No prime divides it

Question 2

Each face of a fair cubical die is labelled with one of the numbers 1 through 6. On every face an arrow is drawn pointing to one of its four neighbouring faces. Suppose these arrows have the property that, beginning at any face and repeatedly moving to the face its arrow indicates, the six moves visit all six faces before returning to the face where you began. In how many different ways can the arrows be drawn so that this property holds?

A 16

B 24

C 32

D 36

E 40

F 48

G 64

H 12

Question 3

Let A and B be subsets of the xy-plane, and consider the following statements.

P: For all real numbers x and y, (x, y) is in A if and only if (x, y) is in B.

Q: There exists a point (x, y) that lies in both A and B.

Which of the following statements is true?

A P is necessary and sufficient for Q.

B P is necessary but not sufficient for Q.

C P is sufficient but not necessary for Q.

D P is not necessary and not sufficient for Q.

E Q is sufficient but not necessary for P.

F P and Q are logically equivalent.

G Q implies P whenever A is non-empty.

H Both P and Q are always true.

Question 4

Jenny is typing all the integers from 1 to ∞, in order. The 5 on her computer is broken however, so she just skips any number containing a 5. What is the 1500th number she types?

A 2044

B 2046

C 2047

D 2048

E 2064

F 1500

G 2144

H 2067

Question 5

Let x and y be real numbers. Which of the following is a necessary and sufficient condition for $|x - y| = |x| + |y|$?

A $x \geq 0$

B $x + y \geq 0$

C $x + y + |x - y| \geq 0$

D $xy \geq 0$

E $x^2 + y^2 - 2xy \geq 0$

F $x - y \geq 0$

G $|x| \geq |y|$

H $xy \leq 0$

Question 6

What is the units digit of 8^{2023}?

A 2

B 4

C 6

D 8

E 0

F 1

G 3

H 9

Question 7

Find the sum of all real x for which

$$2^{3x+1} - 33 \cdot 2^{2x} + 2^{x+3} = 0.$$

A -4

B -3

C 4

D 3

E -2

F 0

G -6

H 2

Question 8

A triangle has altitudes of length 12, 15, and 20. Find its area.

A 120

B 135

C 150

D 180

E 96

F 210

G 144

H 160

Question 9

If p and q are positive integers and $\frac{1000}{1001} < \frac{p}{q} < \frac{1001}{1002}$, what is the minimum value of p?

A 1999

B 2000

C 2001

D 2002

E 2003

F 1001

G 3001

H 2005

Question 10

Let $f(x)$ and $g(x)$ be functions whose derivatives $f'(x)$ and $g'(x)$ exist for all real numbers. Which of the following statements is implied by exactly five of the others?

A $f(x) = g(x)$ for all x in the interval $[0, 2]$.

B f and g are polynomials with $f(k) = g(k)$ for each of $k = 0, 1, 2, 3$.

C $f(x) = g(x + 2) = f(x + 4)$ for all real numbers x.

D $f'(r) = g'(r)$ for some real number r.

E $f(x) > g(x)$ for all real numbers x.

F $f(x) = g(x)$ for all real numbers x.

G f and g are both constant functions.

H None of the statements implies any other.

Question 11

Find the range of values of m so that the equation $3x\sqrt{x} + \frac{m}{x\sqrt{x}} = m - 1$ has two distinct real roots.

A $m > 7 + 4\sqrt{3}$

B $m > 7 - 4\sqrt{3}$

C $m < 7 - 4\sqrt{3}$

D $7 - 4\sqrt{3} < m < 7 + 4\sqrt{3}$

E $m > 1$

F $0 < m < 7 - 4\sqrt{3}$

G $m > 4\sqrt{3}$

H $1 < m < 7 + 4\sqrt{3}$

Question 12

Consider the integral $\int_0^6 \sqrt{6x - x^2}\,\mathrm{d}x$. The exact value of the integral is a. The value of the integral approximated using the trapezoidal rule with 3 equal subintervals is b. Find the value of $|a - b|$.

A $\frac{9\pi}{2} - 8\sqrt{2}$

B $9\pi - 12$

C $\frac{9\pi}{2} - 6\sqrt{2}$

D $9\pi - 8\sqrt{2}$

E $\frac{9\pi}{2} - 4\sqrt{2}$

F $12 - \frac{9\pi}{2}$

G $\frac{9\pi}{2} - 12$

H $\frac{9\pi}{4} - 8\sqrt{2}$

Question 13

Let n be a positive integer and define $I_n = \int_0^1 x^n e^x \, \mathrm{d}x$. Which of the following statements is true?

A $I_1 = 1$

B $I_1 = e - 1$

C The sequence (I_n) is increasing.

D $I_n \to e$ as $n \to \infty$.

E $\frac{e}{n+1} \leq I_n \leq \frac{1}{n+1}$

F $I_1 = e$

G The sequence (I_n) does not tend to 0.

H $I_n \geq 1$ for all n.

Question 14

Let x and y be non-zero numbers. Which of the following must be false if $|x+2| = |y|$?

A $|y| = |x|$

B $y = x^2$

C $|xy| = 4$

D $2(x+2)^2 + 2(y-1)^2 = 1$

E $|y| - |x| = 3$

F $xy < 0$

G $|x| + |y| = 6$

H $y = -x - 2$

Question 15

Simplify the following logarithmic expression:

$$\log\left(\frac{45}{28}\right) + \log\left(\frac{14}{15}\right) + \log\left(\frac{21}{16}\right) - \log\left(\frac{9}{8}\right).$$

A $\log 2$

B $\log \frac{7}{4}$

C $\log \frac{3}{2}$

D 0

E $\log \frac{4}{7}$

F $\log \frac{7}{8}$

G $-\log \frac{7}{4}$

H $\log \frac{9}{8}$

Question 16

The lines ℓ_1 and ℓ_2 are parallel, with ℓ_1 above ℓ_2. A point P lies in the region between them. The segment PA meets ℓ_1 at A, and the acute angle between PA and ℓ_1 has supplement 116°. The segment PB meets ℓ_2 at B, and PB makes an angle of 37° with ℓ_2. The line through P parallel to ℓ_1 and ℓ_2 lies inside the angle APB. Find the size, in degrees, of the angle APB.

A 27

B 79

C 101

D 53

E 116

F 64

G 153

H 37

Question 17

Peter the rabbit likes bounding around the infinite square grid of fields on his farm. Each hop of one field to the north, south, east, or west costs him exactly 1 carrot of energy. One morning he wakes with a store of 120 carrots and keeps hopping between fields until his store is completely used up and he settles down to sleep. In how many different fields could Peter end up falling asleep?

A 14400

B 14641

C 14884

D 7321

E 28800

F 29282

G 14520

H 14762

Question 18

Newton has a wardrobe containing 7 hats of 3 different types: 3 Einstein hats, 2 Euler hats, and 2 Newton hats. He repeatedly takes out 2 hats at a time from the wardrobe at random, and stops if the two hats are of the same type. However, Newton is "Einstein-Euler type-blind", so he also stops if he takes out one Einstein hat and one Euler hat. What is the probability that he stops with 2 hats of the same type? Assume he returns both hats to the wardrobe after each step.

A $\frac{3}{7}$

B $\frac{5}{11}$

C $\frac{5}{21}$

D $\frac{6}{11}$

E $\frac{1}{2}$

F $\frac{5}{16}$

G $\frac{7}{16}$

H $\frac{11}{21}$

Question 19

Find

$$\sum_{k=0}^{\infty} \left\lfloor \frac{1+\sqrt{\frac{8\,000\,000}{4^k}}}{2} \right\rfloor,$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

A 2826

B 2828

C 2830

D 1414

E 2832

F $2000\sqrt{2}$

G 2824

H 2836

Question 20

For each positive integer n, consider the function

$$f_n(x) = (x-2)^n(x+2)^n.$$

Which of the following statements is/are necessarily true?

I. If n is even then $\int_{-2}^{2} f_n(x)\,\mathrm{d}x \leq 34$.

II. For every positive real number k, there exists a positive integer n such that $f_n'(x) > k$ for some real number x with $-1 < x < 0$.

III. If $n > 1$ then there exist real numbers a and b such that the equation $f_n(x) - ax = b$ has exactly four solutions.

A none of them

B I only

C II only

D III only

E I and II only

F I and III only

G II and III only

H I, II and III