TMUA

COMMUNITY PAPERS

Jacqueline Tyler Set A Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved directly from the author-hosted source PDF.

Paper guidance

- Attempt every question and select the best answer from the choices given.
- Diagrams are reproduced directly from the source and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Jacqueline Tyler

1. Find the value of $\int_1^{\square} \frac{2-x^{\square}}{x\sqrt{x}}\,dx$.

A $-\frac{16}{3}$ B $-\frac{10}{3}$ C $-\frac{8}{3}$ D 0 E $\frac{2}{3}$

2. A cuboid has sides of length x, $\sqrt{2}x$ and $2x$ measured in cm.

The volume of the cuboid is numerically equal to twice the total surface area. What is the value of x ?

A 10 B $6+2\sqrt{2}$ C 5 D $\frac{5}{2}$ E $\frac{3+\sqrt{2}}{2}$

3. The function f is given by $f(x) = \dfrac{(2x-1)(3x-2)}{2\sqrt{x^3}}$ find the value of $f'(1)$

A $\dfrac{1}{2}$ B $-\dfrac{1}{2}$ C $\dfrac{7}{4}$ D $\dfrac{19}{4}$ E $\dfrac{13}{2}$

4. What is the sum of all real solutions of the equation $2\sqrt{a} + \frac{7}{\sqrt{a}} = 9 - \frac{6}{a}$

A $4\frac{1}{2}$ B 5 C $12\frac{1}{2}$ D 13 E $13\frac{1}{4}$

5. The line joining the points with coordinates $(p, p-1)$ and $(1-p, 2p)$ is parallel to the line with equation $2x + 3y + 1 = 0$

   What is the value of p ?

   A -1 B $\frac{2}{3}$ C $\frac{5}{4}$ D 2 E 5

6. Anna and Ben are 360 miles apart. Anna travels towards Ben at a constant speed of 25 mph, and Ben travels towards Anna at a constant speed of 65mph. How many miles apart are they, 90 minutes before they meet?

A 40
B 85
C 135
D 225

7. A sequence of terms is given by $a_n = \dfrac{n+1}{n+3}$

What is the product of the first 40 terms?

A $\dfrac{3}{946}$ B $\dfrac{1}{301}$ C $\dfrac{1}{287}$ D $\dfrac{1}{7}$ E $\dfrac{1}{2}$

8. Find the area between the curves given by $y = |x - 1|$ and $y = 3 - |x|$?

A 1 B $\frac{3}{2}$ C 3 D 4 E 6

9. What is the maximum value of the following expression

$$-3x^2 + 12x - 2y^2 - 12y - 39?$$

A -39 B -9 C 0 D 9 E 39

10. A circle is drawn inside a regular hexagon so that the circle touches each side of the hexagon. What fraction of the hexagon is covered by the circle?

A $\frac{\sqrt{3}\pi}{2}$ B $\frac{\sqrt{3}\pi}{6}$ C $\frac{3\sqrt{3}}{2\pi}$ D $\frac{\sqrt{3}}{\pi}$ E $\frac{2\sqrt{3}\pi}{9}$

11. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

A $\int_0^{\frac{\pi}{2}} cosx \ dx$ B $\int_0^{\frac{\pi}{2}} cos^2x \ dx$

C $\int_0^{\frac{\pi}{2}} \sqrt{sinx} \ dx$ D $\int_0^{\frac{\pi}{2}} sin^4x \ dx$ E $\int_0^{\frac{\pi}{2}} 1 - \frac{2}{\pi}x \ dx$

12. $f(x)$ is a quadratic function. Given that the graph of $y = f(x)$ passes through the point $(1, -5)$ and has a turning point at $(-1,3)$, which of the following is an expression for $f(x)$

A $-x^2 - 4x + 1$

B $-2x^2 - 4x + 1$

C $-x^2 - 4x - 3$

D $2x^2 - 4x - 3$

E $-2x^2 + 4x + 1$

13. Find the coefficient of x^3y^2 in the expansion of $(1 + x + y^2)^5$

A 5
B 10
C 20
D 30
E 60

14. Find the complete set of values of x with $\pi < x \leq 2\pi$ such that

$$(\sqrt{3} - 2cosx)\ sinx \geq 0$$

A $\pi \leq x \leq \frac{7\pi}{6}$

B $\pi \leq x \leq \frac{5\pi}{3}$

C $\pi \leq x \leq \frac{11\pi}{6}$

D $\frac{7\pi}{6} \leq x \leq 2\pi$

E $\frac{5\pi}{3} \leq x \leq 2\pi$

F $\frac{11\pi}{6} \leq x \leq 2\pi$

15. The two functions $F(n)$ and $G(n)$ are defined as follows for positive integers *n:*

$$F(n) = \frac{1}{n}\int_0^n (n - \frac{3}{2}x)\ dx \qquad G(n) = \sum_{r=1}^{n} F(r)$$

What is the largest positive integer *n* such that $G(n) < 69$?

A 20 B 21 C 22 D 23 E 24

16. The graph of $y = log_{10}x$ is translated in the negative *y*-direction by 2 units.

This translation is equivalent to a stretch of factor *k* parallel to the *x*-axis?

What is the value of *k*?

A 0.01 B $log_{10}2$ C 0.5 D $log_{2}10$ E 100

17. There are two sets of data: the first set has 10 pieces of data and the second set has 20 pieces of data. The mean of the second set is four more than the mean of the first set.

One of the pieces of data from the first set is exchanged with a piece from the second set. As a result the mean of the second set is now one more than the mean of the first set.

Following this exchange, by how much has the mean of the first set increased?

A 0.5 B 1 C 2 D 4 E 10

18. Find the value of $cos^2 0 + cos^2 2 + cos^2 4 + \ldots + cos^2 86 + cos^2 88 + cos^2 90$

A 0.5 B 1 C 22 D 22.5 E 23

19. The constant c is chosen so that the line $y = c$ divides the region between the graphs of $y = x$ and $y = \sqrt{x}$ into two regions of equal area.

Which of the following equations does c satisfy?

A $6c^4 - 8c^3 + 1 = 0$

B $3c^4 - 4c^3 + 1 = 0$

C $2c^3 - 3c^2 + 1 = 0$

D $4c^3 - 6c^2 + 1 = 0$

20. The curve C has equation $y = x^3 - x^4$

The straight line L is a tangent to C at two distinct points. Find the equation of L.

A $y = \frac{1}{8}x + \frac{1}{64}$

B $y = \frac{1}{8}x + \frac{1}{32}$

C $y = \frac{27}{256}x + \frac{1}{64}$

D $y = \frac{27}{256}x + \frac{1}{32}$