

Jacqueline Tyler Set A Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved directly from the author-hosted source PDF.

Paper guidance

- Attempt every question and select the best answer from the choices given.
- Diagrams are reproduced directly from the source and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Jacqueline Tyler

1. Given that $\cos\theta = -0.6$ and $0 < \theta < 180$ find the value of $\tan\theta$

A $\frac{4}{3}$ B $-\frac{4}{3}$ C $-\frac{3}{4}$ D $\frac{4}{5}$ E $-\frac{\sqrt{34}}{3}$

2. Find the complete set of values of k for which the line $y = 2x - 1$ crosses or touches the curve $y = x^2 + kx + 3$

- A $-2 \leq k \leq 6$
- B $-1 \leq k \leq 3$
- C $-3 \leq k \leq 3$
- D $k \leq -2$ or $k \geq 6$
- E $k \leq -1$ or $k \geq 3$

3. Consider the statement:

No positive integer N that is less than 20 can be written as the sum of two non-prime integers that are greater than 1.

Which of the following is a counterexample to this statement?

I $N = 8$ II $N = 13$ III $N = 24$

A none of them

B I and II only

C II and III only

D I and III only

E I, II and III

4. Five sealed jars, labelled A, B, C, D and E, each contain the same (non-zero) number of £1 coins.

The following statements are attached to the jars:

- A This jar contains less than £4
- B This jar contains £5 or £6
- C This jar contains £1 or £6
- D This jar contains more than £3 and less than £6
- E This jar contains £2 or £3

Exactly one of the jars has a true statement attached to it. Which jar is it?

5. A child is chosen at random from a group. Each child is equally likely to be chosen. Which of the following conditions is/are **necessary** for the probability that the child has a pet to equal exactly $\frac{4}{9}$?

- I Exactly 5 children in the group do not have a pet.
- II The number of children in the group is divisible by 4.
- III The number of children in the group with a pet is divisible by 3.

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

6. What is the smallest positive value of a for which the graph of

$$y = \cos\left(\frac{3}{2}x - 60\right) \text{ has a line of symmetry at } x = a?$$

A 20

B 30

C 40

D 45

E 60

7. The real numbers a, b, c and d satisfy $0 < a < b < c < d$

Which of the following inequalities must be true?

I $a + c < b + d$

II $d - c > b - a$

III $\frac{b}{c} < \frac{d}{a}$

A none of them

B I only

C II only

D III only

E I and II only

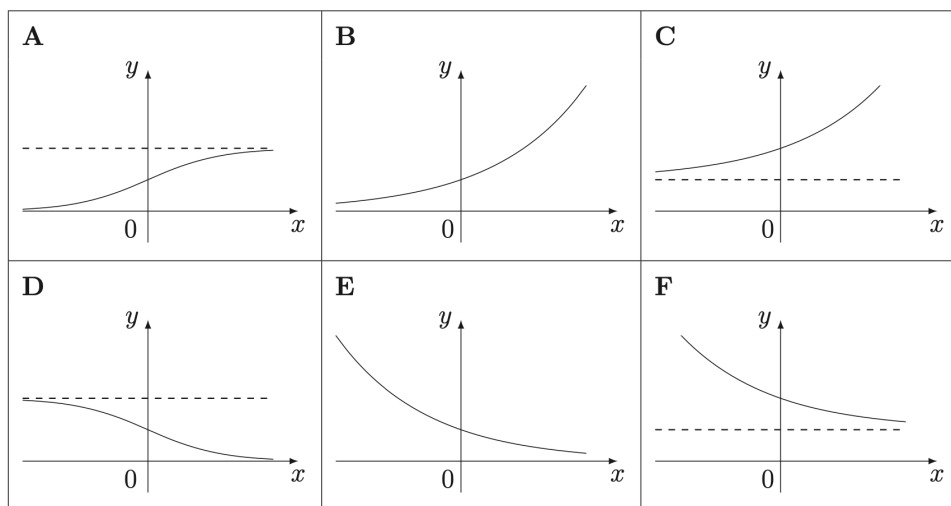
F II and III only

G I and III only

H I, II and III

8. Which one of the following shows the graph of $y = \frac{3}{2^x + 1}$?

(Dotted lines indicate asymptotes.)



9. How many positive solutions for x are there for the equation:

$$\int_0^x (|t - 4| - 3) \, dt = 0$$

- A 0
- B 1
- C 2
- D 4
- E infinitely many

10. Consider the following incorrect attempt at a proof, given that a and b are real numbers such that $a = b$. On which line does the first mistake occur?

- A $a^2 = ab$
- B $a^2 - b^2 = ab - b^2$
- C $(a - b)(a + b) = b(a - b)$
- D $a + b = b$
- E $2b = b$
- F $2 = 1$

11. The function f is defined such that $f(n) = \begin{cases} 0 & \text{if } n \text{ is a multiple of } 5 \\ 1 & \text{if } n \text{ is not a multiple of } 5 \end{cases}$

The function g is defined such that $g(n) = \begin{cases} 0 & \text{if } n \text{ is a multiple of } 7 \\ 1 & \text{if } n \text{ is not a multiple of } 7 \end{cases}$

The function h is defined as $h(n) = (1 - f(n))(1 - g(n))$

What is the value of $h(3) + h(6) + h(9) + \dots + h(2004) + h(2007)$

- A 13
- B 19
- C 38
- D 57
- E 152

12. A sequence is defined by $A_n = A_{n-1} + (-1)^{n+1}(n)^2$

What is the value of $A_{31} - A_{29}$

- A 30
- B 45
- C 57
- D 61
- E 90

13. Consider the following two statements about the polynomial $p(x)$ with exactly two stationary points at $x = a$ and $x = b$ where $a < b$

P: $p(x)$ has a maximum at $x = a$

Q: $p'(x) < 0$ for $a < x < b$

Which of the following is correct?

- A P is **necessary** and **sufficient** for Q
- B P is **not necessary** and **not sufficient** for Q
- C P is **necessary** but **not sufficient** for Q
- D P is **sufficient** but **not necessary** for Q

14. Find the minimum value of $f(x) = \frac{\cos x + 4}{9 + 6\cos x - \sin^2 x}$ for $x \in \mathbb{R}$

A 1

B 3

C $\frac{1}{9}$

D $\frac{1}{3}$

E $\frac{4}{9}$

15. The function $f(x)$ is such that

$$\int_0^3 f(x) \, dx = 8$$

$$\int_{-2}^3 f(x) \, dx = 11$$

$$\int_{-3}^5 f(-x) \, dx = 2$$

Find $\int_{-5}^{-2} f(x) \, dx$

A - 9

B - 6

C - 3

D 3

E 6

16. $f(x) = x^3 + (a + 2)x^2 - 2x - b$ where a and b are positive constants.
Given that $(x - 2)$ and $(x + a)$ are factors of $f(x)$, find the value of $a - b$.

- A - 21
- B - 8
- C 3
- D 8
- E 21

17. Which one of the following is a **necessary and sufficient** condition for the line with equation $y = mx + c$ to be a tangent to the circle with equation $x^2 + y^2 = a^2$

- A $m = 0$ and $c = a$
- B $m = 0$ and $c = \pm a$
- C $m^2 + c^2 = a^2$
- D $c^2 = \frac{a^2}{m^2}$
- E $c^2 = a^2(m^2 + 1)$

18. Let $a, b, c > 0$. The equations: $\log_a b = c$ $a = b^c$

- A specify a , b and c uniquely
- B specify c uniquely but have infinitely many solutions for a and b
- C specify a and b uniquely but have infinitely many solutions for c
- D have no solutions for a , b and c

19. The two circles with equations below touch each other externally.
 $x^2 + y^2 - 4x + 12y + 4 = 0$ and $x^2 + y^2 = a^2$

Find the value of a

- A 6
- B $4\sqrt{2}$
- C $2\sqrt{10} - 6$
- D $2\sqrt{10} + 6$
- E $4\sqrt{2} + 6$

20. Let x be a real number, and consider the inequality $x^2 + 1 \geq 10$.
Which of the following conditions on x is **necessary but not sufficient** for this to be true?

- A $x \leq -3$ or $x \geq 3$
- B $x \neq 0$
- C $x > 10$
- D $x = 3$
- E $x > 0$