

Jacqueline Tyler Set B Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved directly from the author-hosted source PDF.

Paper guidance

- Attempt every question and select the best answer from the choices given.
- Diagrams are reproduced directly from the source and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Jacqueline Tyler

1. For how many values of the non-zero constant k , does the following equation have only one real solution

$$kx^2 - (k - 1)x + k = 0$$

- A no values of k
- B one value of k
- C two values of k
- D all values of k except $k = 1$
- E all values of k

2. How many solutions does the following equation have in the range $0 \leq x \leq \pi$

$$\sin 2x = \cos x$$

- A 1
- B 2
- C 3
- D 4
- E infinitely many

3. For which real numbers x does the following inequality hold

$$\frac{x}{x^2 + 1} \leq \frac{1}{2}$$

- A all real numbers x
- B only for real numbers $x \leq \frac{1}{2}$
- C only for real numbers $x \geq 1$
- D none of the above

4. Consider the following attempt at finding the number of solutions to the equation

$$6 \sin x = 7 \tan x \quad \text{for } 0 \leq x \leq 360.$$

I $6 \sin x = 7 \frac{\sin x}{\cos x}$

II $6 = \frac{7}{\cos x}$

III $\cos = \frac{6}{7}$

IV There are two real solutions to this equation.

Which statement describes this attempt?

- A It is completely correct
- B It is incorrect and the first mistake occurs on line I
- C It is incorrect and the first mistake occurs on line II
- D It is incorrect and the first mistake occurs on line III
- E It is incorrect and the first mistake occurs on line IV

5. The function f is defined for positive integers and satisfies

$$f(1) = 1 \text{ and } f(2n) = f(n) \text{ and } f(2n + 1) = f(n) + 1$$

What is the value of $f(9)$

- A 1
- B 2
- C 3
- D 4
- E 5

6. Consider the following two statements about the polynomial $p(x)$

P: $p(x)$ has at least one real root.

Q: $p(x)$ is a polynomial of order n , where n is an odd integer.

Which of the following is correct?

A P is **necessary** and **sufficient** for Q

B P is **not necessary** and **not sufficient** for Q

C P is **necessary** but **not sufficient** for Q

D P is **sufficient** but **not necessary** for Q

7. The real numbers a and b are such that exactly one of the following statements is true. Which is the true statement?

A $a \geq 0$

B $a < b$

C $a^2 > b^2$

D $|a| \leq |b|$

8. Consider the following statement about the polynomial $p(x)$ where a and b are real numbers with $a < b$.

(*) There exists a number c with $a < c < b$ such that $p(c) = 0$

Which of the following is true?

- A The condition $p(a)p(b) < 0$ is **necessary** and **sufficient** for (*)
- B The condition $p(a)p(b) < 0$ is **not necessary** and **not sufficient** for (*)
- C The condition $p(a)p(b) < 0$ is **necessary** but **not sufficient** for (*)
- D The condition $p(a)p(b) < 0$ is **sufficient** but **not necessary** for (*)

9. A student wishes to evaluate the function $f(x) = \frac{\tan x}{x}$ where x is in radians, but only has a calculator that works in degrees.

What can the student type into their calculator in order to correctly evaluate $f(5)$

- A $\frac{1}{5} \times \tan(\pi \times 5 \div 180)$
- B $5 \times \tan(\pi \times 5 \div 180)$
- C $\tan(\pi \times 5 \div 180) \div (\pi \times 5 \div 180)$
- D $\tan(180 \times 5 \div \pi) \div (180 \times 5 \div \pi)$
- E $\frac{1}{5} \times \tan(180 \times 5 \div \pi)$
- F $5 \times \tan(180 \times 5 \div \pi)$

10. A sequence is such that $u_1 = 6$ and $u_2 = 3$ and $u_{n+1} = \frac{u_n}{u_{n-1}}$ for $n > 1$

What is the value of u_{2023} ?

- A $\frac{1}{2}$
- B $\frac{1}{6}$
- C 2
- D 3
- E 6

11. The fact that $5 \times 6 = 30$,
is a counter example to which of the following statements:
- A the product of any two odd integers is odd
 - B if the product of two integers is a multiple of 4 then the integers are not consecutive
 - C if the product of two integers is not a multiple of 4 then the integers are not consecutive
 - D any even integer can be written as the product of two even integers.

12. What is the reflection of the point (3,4) in the line $3x + 4y = 50$

- A (9 , 12)
- B (12 , 9)
- C (6 , 8)
- D (8 , 6)
- E (12 , 16)
- F (16 , 12)

13. Consider the four options below about a particular statement:

- A The statement is true if $x^2 < 1$
- B The statement is true if and only if $x^2 < 1$
- C The statement is true if $x^2 < 2$
- D The statement is true if and only if $x^2 < 2$

Given that exactly one of these options is correct, which one is it?

14. Find the minimum value of $f(x) = 2x^3 - 9x^2 + 12x + 3$ for $0 \leq x \leq 2$

- A 1
- B 2
- C 3
- D 5
- E 7

15. Given that the numbers x and y satisfy $(x - 1)^2 + y^2 \leq 1$.

What is the largest value that $x + y$ can be?

- A 1
- B $\sqrt{2}$
- C 2
- D $1 + \sqrt{2}$
- E 4

16. Given that $x = -b$ is a root of the equation
 $f(x) = ax^3 + ax^2 + ax + b$ where a and b are constants.
Find the range of possible value of a .

- A There are no possible values of a
- B $a < 1$
- C $0 \leq a \leq \frac{4}{3}$
- D $a \geq 1$
- E a can be any real number

17. Let $a, b, c > 0$. The equations: $\log_b a = 2$ $\log_a(4c - 5) = 2$ $\log_b(c - 1) = 3$

- A specify a uniquely
- B are satisfied by two values of a
- C are satisfied by infinitely many solutions for a
- D are contradictory

18. Which of the following integrals has the largest value?
You are not expected to calculate the exact values of any of these.

A $\int_0^{8\pi} \sin^6 x \, dx$

B $\int_0^{4\pi} 64(\cos^6 x - 1) \, dx$

C $\int_0^1 (x^2 - 2)\sin^6(\pi x) \, dx$

D $\int_0^{4\pi} (3 + \sin x)^6 \, dx$

19. Which of the following statements are true?
- I **There exists** a real y such that **for all** real x , $y > x$
 - II **There exists** a real x such that **for all** real y , $x + y > xy$
 - III **For all** real x , **there exists** real y such that $x - y = xy^2 + 1$
-
- A none of them
 - B I only
 - C II only
 - D III only
 - E I and II only
 - F II and III only
 - G I and III only
 - H I, II and III

20. f is a function and a, b are real numbers.
Given that exactly one of the following statements is true, which one is it?

- A $f(a) \geq f(b)$ if and only if $a \geq b$
- B $f(a) \geq f(b)$ only if $a < b$
- C $f(a) < f(b)$ if $a \geq b$
- D $a \geq b$ if $f(a) \geq f(b)$
- E $a < b$ only if $f(a) \geq f(b)$
- F $a < b$ only if $f(a) < f(b)$