TMUA

COMMUNITY PAPERS

James Chen TMUA Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: James Chen

Question 1

Rick Astley claims that he is never gonna give you up. Which of the following is/are necessary conditions to disprove his claim?

I. He will always give you up. II. There will exist exactly one moment in which he has given you up.

A Neither of them

B I only

C II only

D I and II

Question 2

Vertical lines bisect each of the letters: H,O,T,T,O,G,O. The number of them which are lines of symmetry of the letters is P. Now, vertical lines bisect each of the letters: h,o,t,t,o,g,o. The number of them which are lines of symmetry of the letters is Q. What is the value of $|Q - P|$?

A 0

B 1

C 2

D 3

E 6

F -1

G -2

H -3

I -6

Question 3

Taylor claims to the boy next door: "You belong with me". A popular cheerleader hears the claim and responds: "There exists a moment in which the boy does not belong with you, Taylor". Which option best describes this response?

A The response proves that Taylor's claim is correct

B The response proves that Taylor's claim is incorrect

C The response is the contrapositive of Taylor's claim

D The response is the negation of Taylor's claim

E The response is the converse of Taylor's claim

F The response is the inverse of Taylor's claim

Question 4

How many pairs of integer solutions (x, y) are there to the equation $2^x + 2^y = 160$?

A 0

B 1

C 2

D 3

E 4

F 5

G Infinitely many

Question 5

Only Akshith, James, Pranav, Rujul, and Siddhant are in a room.

The probability that at least two of them share a birthday is P. The probability that James shares a birthday with someone else in the room is Q. The probability that no two of them share a birthday is R.

Which option gives the correct inequality regarding P, Q, and R?

A $P < Q < R$

B $P < R < Q$

C $Q < R < P$

D $Q < P < R$

E $R < P < Q$

F $R < Q < P$

Question 6

Let the sequence u_n be defined such that $u_{n+1} = u_n \sin^{67}(x) \cos^{69}(x)$ and $u_1 = 67$. For what range of values of x, in degrees, does u_n have a convergent sum to infinity?

A No real values of x

B $0 \leq x \leq 45$

C $0 \leq x \leq 67$

D $45 \leq x \leq 67$

E $0 \leq x \leq 136$

F $45 \leq x \leq 136$

G $67 \leq x \leq 136$

H All real values of x

Question 7

How many solutions are there to the equation $e^x \sin x = 1$ with $0 \leq x \leq 2\pi$?

A 0

B 1

C 2

D 3

E 4

Question 8

Consider the claim that $\frac{x+y}{2} > \sqrt{xy}$ for all positive x, y. Here is an attempted proof of this claim:

$(x-y)^2 > 0$ (1)
$x^2 + y^2 - 2xy > 0$ (2)
$x^2 + y^2 + 2xy > 4xy$ (3)
$\frac{(x+y)^2}{4} > xy$ (4)
Taking the square root of both sides, $\frac{x+y}{2} > \sqrt{xy}$ (5)

Which option best describes this attempt to prove the claim?

A The claim is correct and the proof is correct

B The claim is correct but the proof is incorrect

C The claim & proof are incorrect and the first error is on line (1)

D The claim & proof are incorrect and the first error is on line (2)

E The claim & proof are incorrect and the first error is on line (3)

F The claim & proof are incorrect and the first error is on line (4)

G The claim & proof are incorrect and the first error is on line (5)

Question 9

The Subway Sequence is defined as:

$$u_0 = \text{‘She’},\ u_1 = \text{‘Got’},\ u_2 = \text{‘Away’},\ \text{and } u_n = u_{n-3} \text{ for } n \geq 3.$$

What is $u_{(2025+2026+2027+\cdots+2100)}$?

A ‘She’

B ‘Got’

C ‘Away’

D None of the above

Question 10

A student mistakenly believes that $(x+y)^2 = x^2+y^2$. The student attempts to evaluate P^2, where $P = a+b$. The difference between the correct answer and their answer is Q. Which of these quadratics has roots a and b?

A $2x^2 - 2Px + Q$

B $x^2 - Px + Q$

C $2x^2 + Px + Q$

D $x^2 + Px + Q$

E $2x^2 - Px + 2Q$

F $x^2 - 2Px - Q$

G $2x^2 - Px - Q$

H $x^2 - 2Px + 2Q$

Question 11

When setting a TMUA question, I write n statements, where none of them, some of them, or all of them are true. I then ask which of the statements is/are true. What is the minimum number of answer options I must set such that they cover every possible combination (disregarding order) of correct statements?

Note: See Question 1 for an example.

A $\displaystyle\sum_{r=0}^{n}\binom{n}{r}$

B $\displaystyle\sum_{r=1}^{n}\binom{n}{r}$

C $\displaystyle 1+\sum_{r=0}^{n}\binom{n}{r}$

D 2^{n-1}

E 2^{n+1}

F $\displaystyle 1+\sum_{r=1}^{n}\binom{n}{r}$

G $\displaystyle 2+\sum_{r=0}^{n}\binom{n}{r}$

H $\displaystyle 2+\sum_{r=1}^{n}\binom{n}{r}$

Question 12

Which of these numbers is the smallest?

A $67!(69!)^{-1}$

B $69!(67!)^{-1}$

C $6^{\frac{1}{7}}$

D $6^{-\frac{1}{7}}$

E $(68.5)^{-2}$

Question 13

Five students sit the TMUA and achieve integer scores. For their five scores, the following are true:

- There are exactly two different modes.
- The median is not a mode.
- The range is 10.
- The mean is 32.

What is the lowest out of their five scores?

A 25

B 26

C 27

D 28

E 29

F 30

Question 14

The function $f(x)$ is continuous and defined for all real x. Which of the following conditions is/are sufficient for the function to have exactly one root x in the interval $x \in [-1, 1]$?

I. $f(-1) = -67$, $f(2) = 67$, and $f'(x) > 0$ for $x \in [-1, 1]$ II. $f(-2) = 67$, $f(1) = -67$, and $f'(x) < 0$ for all x III. $f(-1) = -67$, $f(1) = 67$

A None of them

B I only

C II only

D III only

E I and II only

F II and III only

G I and III only

H I, II, and III

Question 15

A student attempts to prove, by contradiction, that $\sqrt[n]{2}$ is irrational for all integers $n > 2$. Their proof begins as follows:

$$\text{Assume } \sqrt[n]{2} = \frac{r}{s}, \text{ where } r, s \text{ are integers.}$$

$$2 = \frac{r^n}{s^n}$$

The student despises the idea of infinite descent due to personal trauma or whatever, so shall not use it in their proof. Which of the following theorems is most useful to the student in completing their proof?

A This option knows the right theorem, but is too small to contain it

B $1^n + 2^n + ... + a^n$ is a polynomial in a for integers a and $n > 2$

C $a^n \equiv a \pmod{b}$ when n is an integer greater than 2

D $a^k + b^k = c^k$ has no integer solutions with $k > 2$

Question 16

What is the negation of the following statement?

For all $\epsilon > 0$, there exists $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

A There exists $\epsilon > 0$, such that for all $\delta > 0$, if $0 < |x - a| < \delta$ then $|f(x) - L| \geq \epsilon$.

B There exists $\epsilon > 0$, such that for all $\delta > 0$, there exists $0 < |x - a| < \delta$ such that $|f(x) - L| > \epsilon$.

C There exists $\epsilon > 0$, such that for all $\delta > 0$, there exists $0 < |x - a| < \delta$ such that $|f(x) - L| \geq \epsilon$.

D There exists $\epsilon > 0$, such that for all $\delta > 0$, if $0 < |x - a| < \delta$ then $|f(x) - L| \geq \epsilon$.

E There exists $\epsilon > 0$, such that for all $\delta \leq 0$, if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

F There exists $\epsilon > 0$, such that for all $\delta \leq 0$, there exists $0 < |x - a| < \delta$ such that $|f(x) - L| \geq \epsilon$.

Question 17

Alice and Bob are playing a game of Number Wordle. The answer they are trying to guess is a 5 digit number with no repeated digits. When they make a guess, a digit turns gray if it is not present in the answer, green if it is in the correct position, and yellow if it is in the answer but not the correct position. Alice guesses 67041 and then Bob guesses 23589, and the results are shown below:

How many possible answers are there?

A 1

B 2

C 5

D 6

E 7

F 8

G 9

H 10

I 11

J 12

Question 18

The cubic polynomial $x^3 - ax^2 + bx - c$ is equivalent to $(x-a)(x-b)(x-c)$, where a, b, c are real non-zero constants. What is the complete set of possible values of b?

A 1

B -1

C 1 or -1

D Any positive number

E Any negative number

F All real numbers

G No real numbers

H Any number greater than a and less than c

I Any number less than a and greater than c

J $pa + qc$, where p and q are integers

Question 19

An equilateral triangle with side-length 1 is drawn, and 5 points are plotted in or on the triangle. What is the minimum value of d such that at least two points must be no further than d apart, no matter the configuration of the points?

A 0.25

B $\frac{\sqrt{2}}{2}$

C 1

D $\frac{\sqrt{3}}{2}$

E 0.5

F $\sqrt{2}$

G $\sqrt{3}$

Question 20

The equation $5^2 - 5(2x^2 + 1) + x^4 - x = 0$ has four solutions, two of which can be written as $\frac{1}{Q}(-1 \pm \sqrt{P})$ and the other two of which can be written as $\frac{1}{2}(R \pm \sqrt{S})$ for integer constants P, Q, R, S. What is the value of $P + Q + R + S$?

A 37

B 38

C 39

D 40

E 41

F 42