TMUA

COMMUNITY PAPERS

Juan Set 1 Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 25 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Juan

Question 1

The numbers $2, 3, 4, 5, 6, 7, 8$ and 9 are to be placed into the boxes below, one different number per box, so that all the equalities hold.

$$\square \times \square = 12$$
$$A \div \square = 2$$
$$\square + \square = 12$$
$$B - \square = 2$$

What is $A + B$?

A 13

B 14

C 15

D 16

E 17

Question 2

Let a, b, and c be distinct digits such that

$$a \cdot b < 45 < b \cdot c < 50 < a \cdot c$$

What is $a + b + c$?

A 17

B 18

C 19

D 20

E 21

Question 3

Let a, b, c, d be positive integers, and let

$$M = 6^a \cdot 5^b, \qquad N = 10^c \cdot 9^d$$

be positive integers such that

$$\gcd(M, N) = 2^3 \cdot 3^2 \cdot 5, \qquad \operatorname{lcm}(M, N) = 2^5 \cdot 3^3 \cdot 5^5$$

What is $a + b + c + d$?

A 8

B 9

C 10

D 11

E 12

Question 4

Five distinct prime numbers have a sum equal to 100, and their product is equal to the six-digit natural number $ABCABC$ (where the digits are A, B, C — this does not represent a product).

What is $A + B + C$?

A 8

B 11

C 14

D 17

E 20

Question 5

Let x be a real number different from -1, 0, and 1. Consider the set

$$\left\{x^3,\ x^2,\ x,\ -x,\ -\frac{1}{x}\right\}$$

When the elements of this set are ordered from smallest to largest, which element is never located exactly in the middle?

A x^3

B x^2

C x

D $-x$

E $-\frac{1}{x}$

Question 6

Let a and b be positive real numbers, and consider the equations

$$x^2 + ax + b = 0, \qquad ax^2 + (b+3)x + a = 0$$

Given that the solution set of each of these equations has exactly 1 element, what is the product of the distinct possible values of $a + b$?

A 24

B 32

C 45

D 72

E 120

Question 7

Let $P(x)$ and $Q(x)$ be polynomials with real coefficients such that $P(x)+Q(x)$ is a second-degree polynomial, and

$$P(x) \cdot Q(x) = -4 \cdot (x-1)^4 \cdot (x-2)^2, \qquad P(3) = -16$$

What is the value of $Q(4)$?

A 12

B 24

C 36

D 48

E 54

Question 8

For an arithmetic sequence (a_n), it is given that

$$a_1 \cdot a_2 \cdot a_3 = 2, \qquad a_2 \cdot a_3 \cdot a_4 = 14$$

What is the product $a_3 \cdot a_4 \cdot a_5$?

A 28

B 35

C 42

D 49

E 56

Question 9

Let n be a positive integer. In the expansion of

$$(x^2 + x)^n$$

the coefficient of the term with x^{19-n} and the coefficient of the term with x^{16-n} are both equal to the positive integer k.

What is k?

A 6

B 12

C 15

D 18

E 21

Question 10

Sena has chosen a series A consisting of three films and a series B consisting of another three films, so that she can watch a different film on each of six specific evenings. For each series, she will not watch the second film of that series before the first, and will not watch the third film of that series before the second.

In how many different orders can Sena decide to watch these six films?

A 20

B 24

C 27

D 30

E 32

Question 11

A shopping mall has two entrance gates. In front of the first gate there are 2 parking lots named Blue and Red; in front of the second gate there are 3 parking lots named Yellow, Orange, and Green. Kartal, who comes to this mall, arrives at a randomly chosen gate and parks his car in a randomly chosen lot in front of that gate before entering the mall. When leaving the mall, having forgotten which gate he entered from and in which lot he parked, he exits from a randomly chosen gate and searches for his car in a randomly chosen lot in front of that gate.

What is the probability that the lot in which Kartal searches for his car is the lot in which he actually parked it?

A $\frac{1}{5}$

B $\frac{5}{24}$

C $\frac{6}{25}$

D $\frac{7}{36}$

E $\frac{11}{48}$

Question 12

Let a and b be non-consecutive positive integers such that

$$\ln(a!) = \ln(b!) + 3\ln 2 + 2\ln 3 + \ln 7$$

What is $a + b$?

(Note that $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$)

A 10

B 13

C 15

D 18

E 20

Question 13

Let a be a nonzero real number and b, c be real numbers. For the function f defined on the set of real numbers by $f(x) = ax + b$, and its inverse function f^{-1}, it is given that

$$\lim_{x \to b} \frac{f(x)}{f^{-1}(x)} = c, \qquad f(1) = 3$$

What is the sum of the distinct possible values that c can take?

A 6

B 7

C 10

D 11

E 12

Question 14

Let a and b be real numbers. The function f, defined on the set of real numbers by

$$f(x) = \begin{cases} x^2 - ax + 6, & x \leq a \\ 2x + a, & a < x \leq b \\ 11 - 2x + b, & x > b \end{cases}$$

is known to be continuous on its domain.

What is the product $a \cdot b$?

A 4

B 6

C 8

D 10

E 12

Question 15

Let n be a positive integer and a a nonzero real number. For a polynomial function f of degree n with leading coefficient a, it is given that

$$\left((f(x))^3\right)' = (f'(x))^4$$

What is the product $a \cdot n$?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D 1

E 2

Question 16

In the coordinate plane, for the function $y = f(x)$, it is given that:

- the tangent line to the graph at the point $(2, f(2))$ is $y = 3x - 1$,
- the tangent line to the graph at the point $(5, f(5))$ is $y = 2x + 4$.

For the function g defined by

$$g(x) = x^2 \cdot (f(f(x)))$$

what is the value of $g'(2)$?

A 64

B 72

C 80

D 88

E 96

Question 17

Let a be a positive real number such that

$$\int_1^4 (a\sqrt{x} + x\sqrt{a})\ dx = \frac{59}{3}a$$

What is a?

A $\frac{1}{4}$

B $\frac{1}{9}$

C $\frac{1}{16}$

D $\frac{1}{25}$

E $\frac{1}{36}$

Question 18

Let f be a function defined and continuous on the set of real numbers, and let g be defined by $g(x) = 2x + 2$. It is given that

$$\int_{-1}^{1} f(g(x))\,dx = 18, \qquad \int_{2}^{4} g(f(x))\,dx = 18$$

What is the value of the integral $\displaystyle\int_{0}^{2} f(x)\,dx$?

A 20

B 23

C 26

D 29

E 32

Question 19

Which of the following is the simplified form of the expression below?

$$\frac{1-\cos(4x)}{\sin(4x)+2\sin(2x)}$$

A $\sin x$

B $\tan x$

C $\cot x$

D $\sec x$

E $\operatorname{cosec} x$

Question 20

Let x, y, z be distinct elements, one each, of the set $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\right\}$, such that

$$\sin x < \tan y < \sec z$$

Which of the following is the correct ordering of x, y, and z?

A $x < y < z$

B $y < x < z$

C $y < z < x$

D $z < x < y$

E $z < y < x$

Question 21

Bonus question. Let a triangle ABC have side lengths $|BC| = a$ units, $|AC| = b$ units, and $|AB| = c$ units, satisfying

$$2a^2 = 2b^2 + 2c^2 + 3bc$$

Given that x is the size of angle BAC, what is the value of $\tan x$?

A $-\frac{\sqrt{2}}{3}$

B $-\frac{\sqrt{3}}{3}$

C $-\frac{\sqrt{5}}{3}$

D $-\frac{\sqrt{6}}{3}$

E $-\frac{\sqrt{7}}{3}$

Question 22

Bonus question. Let $0 < x < \dfrac{\pi}{2}$ satisfy

$$2\cos^2 x + 9\sin^2 x + 2\sin(2x) = 9$$

What is the value of $\cot x$?

A $\dfrac{4}{7}$

B $\dfrac{7}{6}$

C $\dfrac{3}{5}$

D $\dfrac{2}{3}$

E $\dfrac{5}{2}$

Question 23

Bonus question. Let a and b be positive real numbers. In the coordinate plane, the lines $y = -\sqrt{3}x$ and $y = ax + b$, together with the x-axis, enclose an equilateral-triangle-shaped region whose area is $9\sqrt{3}$ square units.

What is the product $a \cdot b$?

A 18

B 24

C 27

D 30

E 36

Question 24

Bonus question. A circle drawn in the coordinate plane is known to satisfy:

- it has exactly one common point with the line $d_1 : y - \frac{4x}{3} - 46 = 0$,
- it has exactly two common points with the line $d_2 : y - \frac{4x}{3} - 6 = 0$,
- it has no common point with the line $d_3 : y - \frac{4x}{3} - 1 = 0$.

Which of the following could be the radius of this circle, in units?

A 11

B 13

C 15

D 17

E 19

Question 25

Bonus question. In the coordinate plane, translating a point A by 15 units in the negative direction along the x-axis produces a point that lies on the line

$$d : 4x - 3y + 24 = 0$$

By how many units must point A be translated in the positive direction along the y-axis so that the resulting point lies on the line d?

A 9

B 12

C 16

D 20

E 25