TMUA

COMMUNITY PAPERS

Juan Set 2 Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 26 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Juan

Question 1

Let m and n be real numbers. In the coordinate plane, the circle passing through the point $A(4, 1)$ is given by

$$x^2 + y^2 - 2x + 6y = n$$

The line $y = mx$ drawn in the plane intersects this circle at points B and C.

Given that angle BAC is a right angle, what is $m + n$?

A 8

B 9

C 10

D 11

E 12

Question 2

For digits A, D, T, and Y, the following addition is given:

$$\begin{array}{r} TYT \\ AYT \\ +\ YDT \\ \hline 2024 \end{array}$$

where $TYT < AYT$.

What is the product $Y \cdot D \cdot T$?

A 48

B 64

C 80

D 96

E 112

Question 3

Let a, x, y be positive real numbers such that

$$-2x^2 + y^2 = 2a, \qquad 3x^2 - 2y^2 = -6a$$

What is the ratio $\dfrac{y}{x}$?

A 1

B $\sqrt{2}$

C $\sqrt{3}$

D 2

E 3

Question 4

Let a be a digit such that the remainder when 25! is divided by $(23! - a)$ is 60^2.

What is a? (Note that $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$)

A 2

B 3

C 4

D 5

E 6

Question 5

Let x, y, z be distinct prime numbers such that

$$x + y \cdot z = 21$$

What is $x + y + z$?

A 12

B 14

C 16

D 18

E 20

Question 6

Let a be an integer such that the inequality

$$0 < \left|x^2 - 2x + 2\right| - x^2 - x < a$$

is satisfied by exactly 4 integer values of x.

What is the sum of the distinct possible integer values of a?

A 33

B 36

C 39

D 42

E 45

Question 7

Let a and b be positive real numbers such that for each of the equations

$$ax^2 - 2x + b = 0, \qquad bx^2 - 3bx + a = 0$$

the sum of the roots is 1 more than the product of the roots.

Which of the following could be the second-degree equation whose roots are a and b?

A $9x^2 + 8x + 18 = 0$

B $9x^2 - 14x + 8 = 0$

C $9x^2 - 18x + 14 = 0$

D $9x^2 - 8x + 14 = 0$

E $9x^2 - 18x + 8 = 0$

Question 8

Let a and b be real numbers, and let the function f, defined on the set of real numbers by

$$f(x) = x^3 + 9x^2 + ax + b$$

take positive values for positive real numbers and negative values for negative real numbers.

What is the smallest integer value that a can take?

A 9

B 13

C 17

D 21

E 25

Question 9

For an arithmetic sequence (a_n) with common difference r, it is given that

$$a_{30} - a_{25} < 53, \qquad a_{25} - a_8 > 53$$

What is the sum of the possible integer values of r?

A 49

B 56

C 64

D 72

E 84

Question 10

Let a, x, y be positive real numbers such that, arranged from smallest to largest,

$$\log_a x, \quad \log_a y, \quad \log_a (x+y)$$

are consecutive integers.

What is the value of $\log_a(2a+1)$?

A -2

B -1

C 2

D 3

E 4

Question 11

Melisa is going to stack, on a table, 10 circular pieces of cardboard whose radii are 1 cm, 2 cm, 3 cm, ..., 10 cm (each a different natural number of centimetres), one on top of another with their centres aligned.

In how many different ways can Melisa arrange this stack so that, viewed from above, exactly one of the circles is completely hidden from view?

A 36

B 40

C 45

D 48

E 54

Question 12

A certain bus arrives at the stop near Duru's house at exactly 09:02 with probability $\frac{7}{10}$, and at exactly 09:03 with probability $\frac{3}{10}$. Duru leaves her house at exactly 09:00 to catch this bus; the time it takes her to reach the stop is 100 seconds with probability $\frac{1}{2}$, 150 seconds with probability $\frac{3}{10}$, and 250 seconds with probability $\frac{1}{5}$.

What is the probability that Duru is at the stop at the moment the bus arrives?

A $\frac{55}{100}$

B $\frac{59}{100}$

C $\frac{63}{100}$

D $\frac{67}{100}$

E $\frac{71}{100}$

Question 13

Let m and n be natural numbers such that the constant term in the expansion of

$$\left(x + \frac{5}{x^m}\right)^n$$

is 60.

What is $m + n$?

A 36

B 35

C 31

D 27

E 23

Question 14

Let a and b be real numbers such that, in the coordinate plane, the parabola $y = x^2 + ax + b$ is tangent to the x-axis and to the line $y = x$.

What is the product $a \cdot b$?

A $\frac{1}{2}$

B $\frac{1}{4}$

C $\frac{1}{8}$

D $\frac{1}{16}$

E $\frac{1}{32}$

Question 15

Let a and b be real numbers, and let the function f, defined by

$$f(x) = ax^3 + bx^2 + x + 7$$

be strictly increasing everywhere. Given that $f(-1) = 0$, what is the sum of the distinct possible integer values of b?

A 11

B 13

C 15

D 17

E 19

Question 16

Let k and m be real numbers, and let the functions f and g be defined on the set of real numbers by

$$f(x) = 2x^3 - 9x^2 - mx - k, \qquad g(x) = x^3 \cdot f(x)$$

Both f and g have a local extremum at $x = -1$.

What is $k + m$?

A 31

B 33

C 35

D 37

E 39

Question 17

Let f and g be differentiable functions defined on the set of real numbers satisfying

$$\int_1^2 f'(3x)\,dx = 4, \qquad \int f(2x)\,dx = g(x) + C \ \ (C \text{ constant})$$

Given that $f(3) = 5$, what is the value of $g'(3)$?

A 1

B 5

C 9

D 13

E 17

Question 18

Let

$$a = \sin(40^\circ), \qquad b = \sec(40^\circ), \qquad c = \tan(40^\circ)$$

Which of the following is the correct ordering of a, b, and c?

A $a < b < c$

B $a < c < b$

C $b < a < c$

D $b < c < a$

E $c < a < b$

Question 19

Which of the following is the simplified form of the expression

$$\frac{\cos(2x+y)+\sin(2x-y)}{\cos(2x)+\sin(2x)}$$

A $\cos y - \sin y$

B $\cos y + \sin y$

C $\cos x - \sin y$

D $\sin x - \cos y$

E $\sin x - \cos x$

Question 20

Let $0 < a < \dfrac{\pi}{2}$ satisfy

$$\cos^2 a - \cos(2a) = \sin(2a)$$

Which of the following is true for this value of a?

A $\tan a = \dfrac{1}{5}$

B $\cot a = \dfrac{2}{\sqrt{5}}$

C $\cos a = \dfrac{1}{\sqrt{5}}$

D $\operatorname{cosec} a = \sqrt{5}$

E $\sin(2a) = \dfrac{3}{5}$

Question 21

Bonus question. Let $0 \leq x \leq \pi$ satisfy

$$\sqrt{2}\sin(4x) - \cos(8x) = 1$$

What is the sum of the values of x that satisfy this equation?

A $\dfrac{\pi}{3}$

B $\dfrac{3\pi}{4}$

C π

D $\dfrac{3\pi}{2}$

E 2π

Question 22

Bonus question. The altitude of a triangle is a straight line drawn from a corner vertex down to the opposite side so that it hits that side at a right angle.

In a triangle ABC, the length of side AB is equal to half the length of side BC. Given that the lengths of two of this triangle's altitudes are 4 units and 10 units, which of the following could be the length of the third altitude?

I. 2 units

II. 5 units

III. 8 units

A Only I

B Only II

C Only III

D I and II

E II and III

Question 23

Bonus question. The centroid of a triangle can be calculated by taking the mean of the x-coordinates and the y-coordinates of its vertices.

In the coordinate plane, the points $A(9, 2)$, $B(10, 1)$, C, $D(4, 13)$, $E(3, 6)$, and F are given.

Given that triangle ABC and triangle DEF have the same centroid, what is the distance between points C and F, in units?

A 10

B 13

C 15

D 17

E 20

Question 24

Bonus question. In the coordinate plane, what is the area, in square units, of the triangular region bounded by the lines

$$2x - y = 0, \qquad x + 2y = 0, \qquad x - 8y + 30 = 0$$

A 9

B 12

C 15

D 18

E 21

Question 25

Bonus question. In the coordinate plane, let C be the reflection of the point $A(a, b)$ about the point $B(3, 0)$, and let D be the reflection of A about the y-axis.

Given that the line through C and D has the equation $y = -x - 1$, what is $a + b$?

A 7

B 13

C 15

D 19

E 24

Question 26

Bonus question. In the coordinate plane, the point $A(11, 9)$ lies in the interior of a circle that is tangent to the line $y = x$ at the point $B(7, 7)$.

What is the smallest integer value, in units, that the radius of this circle can take?

A 6

B 8

C 10

D 12

E 14