TMUA

COMMUNITY PAPERS

Maths Made Easy TMUA Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Maths Made Easy

Question 1

Real numbers x and y satisfy

$$x^2 + xy + y^2 = 7.$$

Find the complete set of values that $x + y$ can take.

A $-2\sqrt{3} \leq x + y \leq 2\sqrt{3}$

B $-\sqrt{7} \leq x + y \leq \sqrt{7}$

C $-\frac{2\sqrt{21}}{3} \leq x + y \leq \frac{2\sqrt{21}}{3}$

D $x + y$ can be any real number

E $-\sqrt{21} \leq x + y \leq \sqrt{21}$

Question 2

A geometric progression has first term a and common ratio r. The 1st, 2nd, and 4th terms form an arithmetic progression (in that order).

Given $r \neq 1$ and the geometric series converges, find the sum to infinity of the geometric series in terms of a.

A $a(\sqrt{5}-1)$

B $a\left(\frac{\sqrt{5}+1}{2}\right)$

C $a\left(\frac{\sqrt{5}-1}{2}\right)$

D $\frac{a(3+\sqrt{5})}{2}$

E $\frac{a(\sqrt{5}+3)}{4}$

Question 3

How many solutions does the equation

$$4\sin^4 x - 3\sin^2 x + \sin^2 x\cos^2 x = 0$$

have in the interval $0 \leq x \leq 2\pi$?

A 3

B 4

C 5

D 6

E 7

Question 4

Find the smallest positive integer n for which

$$\sum_{k=1}^{n} \log_3 \left(1 + \frac{1}{k}\right) > 3.$$

A 24

B 25

C 26

D 27

E 28

Question 5

A circle of radius r has its centre on the positive y-axis and is tangent to both the parabola $y = x^2$ and the straight line $y = x + 1$. Find r.

A $\frac{\sqrt{5}-\sqrt{2}}{2}$

B $\frac{\sqrt{3}-1}{2}$

C $\sqrt{5} - \sqrt{2}$

D $\frac{\sqrt{6}-\sqrt{3}}{3}$

E $\frac{\sqrt{10}-\sqrt{5}}{4}$

Question 6

For real numbers x and y with $xy = 1$, find the maximum value of

$$\frac{(x+y)^2}{x^2+y^2}.$$

A 1

B $\frac{3}{2}$

C 2

D 3

E 4

Question 7

Let $u_{n+1} = u_n^2 - 2$ with $u_1 = t$.

For how many integer values of t does there exist some positive integer N such that $u_N = 2$?

A 1

B 2

C 3

D 4

E Infinitely many

Question 8

Positive real numbers a, b, c satisfy $ab + bc + ca = 1$.

What is the greatest possible value of

$$\frac{a+b+c}{(1+a)(1+b)(1+c)}?$$

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{\sqrt{3}}{2}$

E 1

Question 9

For positive real numbers x, y define

$$S = \frac{x}{y} + \frac{y}{x} + x + y.$$

Which is the greatest lower bound of S?

A 4

B $2 + 2\sqrt{2}$

C 6

D $3 + 2\sqrt{3}$

E 8

Question 10

Let $a_1 = 1$ and $a_{n+1} = a_n + \frac{1}{a_n}$.

Which statement is true?

A $a_n < \sqrt{2n}$ for all n

B $a_n > \sqrt{2n}$ for all n

C $a_n = \sqrt{2n}$ for infinitely many n

D $a_n < \sqrt{2n+1}$ for all n

E $a_n > \sqrt{2n+1}$ for all n

Question 11

The polynomial $f(x) = x^3 + cx^2 + dx - 6$ satisfies both of the following:

- $(x + 2)$ is a factor of $f(x)$;
- $f(x)$ leaves a remainder of 3 when divided by $(x - 1)$.

Once c and d are determined, $f(x)$ can be written as $(x+2)q(x)$ for some quadratic $q(x)$. Find the product of the roots of $q(x) = 0$.

A -6

B -3

C -2

D 3

E 6

Question 12

Find the coefficient of x^3 in the expansion of $(2-x)^3(1+x)^5$.

A -55

B -31

C -11

D 11

E 31

Question 13

n distinct points lie within a unit square. Given that at least two points lie less than $\frac{\sqrt{2}}{3}$ away from each other, find the minimum n.

A 8

B 9

C 10

D 19

E 20

Question 14

Find the number of integer solutions to

$$|x| + |y| \leq 400$$

A 80,100

B 320,801

C 801

D 160,801

E 320,000

Question 15

The lengths of a triangle's sides follow a geometric progression (common ratio r). It follows that:

A $1 < r < \frac{1+\sqrt{5}}{2}$

B $0 < r < \frac{1+\sqrt{5}}{2}$

C $0 < r < \frac{1-\sqrt{5}}{2}$

D $\frac{2}{1+\sqrt{5}} < r < \frac{1+\sqrt{5}}{2}$

E $\frac{1+\sqrt{5}}{2} < r < 2$

Question 16

What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $||x| - 2| + ||y| - 2| \leq 4$?

A 24

B 32

C 64

D 96

E 112

Question 17

For real x, find the number of solutions to

$$\sin(\pi \cos x) = \cos(\pi \sin x).$$

A 4

B 6

C 8

D 10

E 12

Question 18

A pyramid has a square base of side length s and four equilateral triangular faces. A sphere of radius r is inscribed in the pyramid, touching all five faces. A second sphere of radius R is circumscribed about the pyramid, passing through all five vertices.

Find the ratio of the volume of the circumscribed sphere to the inscribed sphere.

A $6 + 10\sqrt{3}$

B $8 + 4\sqrt{3}$

C $10 + 6\sqrt{3}$

D $12 + 5\sqrt{3}$

E $16 + 2\sqrt{3}$

Question 19

A rectangle is inscribed in a circle of radius R, so that all four vertices lie on the circle. Find the maximum possible area of the rectangle.

A R^2

B $\sqrt{2}R^2$

C $2R^2$

D $2\sqrt{2}R^2$

E $4R^2$

Question 20

Two chords are chosen independently and uniformly at random within a unit circle. The probability that a single random chord in a unit circle has length greater than $\sqrt{3}$ is P. Given that the two chords intersect inside the circle, find the probability that both chords have a length exceeding $\sqrt{3}$.

A $\frac{1}{9}$

B $\frac{1}{12}$

C $\frac{\pi-2}{4\pi}$

D $\frac{1}{18}$

E $\frac{1}{6}$