TMUA

COMMUNITY PAPERS

Nabla Tuition TMUA Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Nabla Tuition

Question 1

A sequence is defined by

$$x_1 = 1, \qquad x_{n+1} = \frac{1}{1 + x_n} \quad (n \geq 1).$$

What is the value of x_6?

A $\frac{3}{5}$

B $\frac{5}{8}$

C $\frac{8}{13}$

D $\frac{13}{21}$

E $\frac{21}{34}$

Question 2

The graphs

$$y = x^2 - 4x + c \qquad \text{and} \qquad y = 2x - 3$$

touch at exactly one point. Find c.

A 3

B 4

C 5

D 6

E 7

Question 3

The line

$$y = mx + 3$$

intersects the parabola $y = x^2$ at two points A and B. The horizontal distance between A and B is 4. Find m^2.

A 1

B 2

C 3

D 4

E 6

Question 4

The function f is defined by

$$f(x) = \frac{x-1}{x+1}, \qquad x \neq -1.$$

Find the solution of

$$f(f(f(x))) = 2.$$

A -3

B $-\frac{1}{3}$

C 0

D $\frac{1}{3}$

E 3

Question 5

An acute angle θ satisfies

$$\sin\theta = 2\cos^2\theta.$$

Find $\tan^2\theta$.

A $\frac{\sqrt{17}-3}{2}$

B $\frac{\sqrt{17}-1}{4}$

C $\frac{\sqrt{17}+1}{4}$

D $\frac{\sqrt{17}-1}{2}$

E $\frac{\sqrt{17}+1}{2}$

Question 6

Find the coefficient of x^5 in

$$(1+x)^4(1+x^2)^3.$$

A 12

B 18

C 20

D 24

E 30

Question 7

A positive real number x satisfies

$$x + \frac{1}{x} = 3.$$

Find

$$\frac{x^2 + 1}{x^2 - x + 1}.$$

A 1

B $\frac{4}{3}$

C $\frac{3}{2}$

D 2

E 3

Question 8

The tangent to the curve $y = x^2$ at the point (a, a^2), where $a > 0$, meets the coordinate axes at P and Q. If $OP = OQ$, where O is the origin, find a.

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D 2

E 4

Question 9

The tangent to the circle

$$x^2 + y^2 = 25$$

at the point $(3, 4)$ meets the coordinate axes at A and B. Let O be the origin.

What is the area of triangle OAB?

A $\frac{25}{2}$

B $\frac{125}{8}$

C $\frac{625}{24}$

D $\frac{125}{4}$

E $\frac{625}{12}$

Question 10

The circle

$$(x-2)^2+(y+1)^2=9$$

is tangent to each of the two lines of the form

$$y=x+c.$$

What is the sum of the two possible values of c?

A -8

B -6

C -4

D 4

E 6

Question 11

How many solutions are there to

$$2\sin^2(2x) - 3\sin(2x) + 1 = 0$$

in the interval $0 \leq x \leq 2\pi$?

A 4

B 5

C 6

D 7

E 8

Question 12

A four-digit number is formed by choosing four distinct digits from

$$1, 2, 3, 4, 5, 6$$

and arranging them at random.

What is the probability that the number is even and greater than 4000?

A $\frac{1}{5}$

B $\frac{7}{30}$

C $\frac{1}{4}$

D $\frac{3}{10}$

E $\frac{1}{3}$

Question 13

The equation

$$4^x - a\,2^x + 1 = 0$$

has two real solutions whose difference is 2.

Given that $a > 0$, find a.

A 2

B $\frac{5}{2}$

C 3

D $\frac{7}{2}$

E 4

Question 14

An infinite geometric series has positive first term and common ratio r, where

$$0 < r < 1.$$

The sum of the terms in odd-numbered positions is 14, and the sum of the terms in positions $2, 5, 8, ...$ is 6.

What is r?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Question 15

The tangent to the curve

$$y = \frac{1}{x}, \qquad x > 0,$$

at the point whose x-coordinate is t meets the positive coordinate axes at A and B.

What is the least possible value of $OA + OB$?

A 2

B $2\sqrt{2}$

C 3

D 4

E $4\sqrt{2}$

Question 16

Three distinct integers are chosen at random from

$$1, 2, 3, 4, 5, 6, 7, 8.$$

What is the probability that their sum is divisible by 3?

A $\frac{2}{7}$

B $\frac{5}{14}$

C $\frac{3}{8}$

D $\frac{2}{5}$

E $\frac{3}{7}$

Question 17

The curve

$$y = x - x^3$$

lies above the x-axis for $0 < x < 1$.

The line $x = a$, where $0 < a < 1$, divides the area between the curve and the x-axis from $x = 0$ to $x = 1$ into two equal parts.

What is the value of a^2?

A $1 - \frac{1}{\sqrt{2}}$

B $\frac{1}{\sqrt{2}}$

C $\sqrt{2} - 1$

D $\frac{1}{2}$

E $2 - \sqrt{2}$

Question 18

A right circular cylinder is inscribed in a sphere of radius 3. The axis of the cylinder passes through the centre of the sphere. What is the greatest possible volume of the cylinder?

A $6\sqrt{3}\pi$

B $9\sqrt{3}\pi$

C $12\sqrt{3}\pi$

D 18π

E 24π

Question 19

The equation

$$x^3 - 3x = k$$

has three distinct real solutions $p < q < r$. For how many integer values of k do all three solutions lie in the interval $(-2, 2)$?

A 1

B 2

C 3

D 4

E 5

Question 20

For a real constant k, the function f is defined by

$$f(x) = x^3 - 3x + k.$$

The equation

$$f(f(x)) = k$$

has exactly seven distinct real solutions.

Which of the following gives the complete set of possible values of k?

A $-2 < k < 2$

B $-2 < k < \sqrt{3} - 2$ or $2 - \sqrt{3} < k < 2$

C $-2 < k \leq \sqrt{3} - 2$ or $2 - \sqrt{3} \leq k < 2$

D $\sqrt{3} - 2 < k < 2 - \sqrt{3}$

E $k < \sqrt{3} - 2$ or $k > 2 - \sqrt{3}$