TMUA
COMMUNITY PAPERS

Qiuyun TMUA Simulation Test Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Qiuyun

Question 1

The coefficient of x^5 in the expansion of $(ax^2 + 3x + 2)^3$ is 81.

Find all possible values of the constant a.

A ± 9

B ± 6

C ± 3

D ± 1

E There are no possible values of a.

Question 2

Evaluate

$$\int_1^8 \left(\frac{\sqrt[3]{x}-1}{\sqrt[3]{x}}\right)^2 \, \mathrm{d}x$$

A -1

B 0

C 1

D 2

E 3

Question 3

Given a geometric sequence $\{a_n\}$, we have the first term $a_1 = \frac{4}{3}$ and the common ratio $q = \frac{4}{3}$.

Which of the following statements is true about the sequence $\{\ln(a_n)\}$?

A $\{\ln(a_n)\}$ is a geometric sequence and the common ratio is $\ln\left(\frac{4}{3}\right)$.

B $\{\ln(a_n)\}$ is an arithmetic sequence and the common difference is $\frac{4}{3}$.

C $\{\ln(a_n)\}$ is an arithmetic sequence and the common difference is $\ln\left(\frac{4}{3}\right)$.

D $\{\ln(a_n)\}$ is a geometric sequence and the common ratio is $\frac{4}{3}$.

E $\{\ln(a_n)\}$ is neither an arithmetic sequence nor a geometric sequence.

Question 4

Consider the polynomial $P(x) = x^4 - 6x^3 + 11x^2 - 7x + 1$.

Which of the following statements is correct?

A $x - 1$ is a factor of $P(x)$, and the quotient is $x^3 - 5x^2 + 6x - 1$.

B $x + 1$ is a factor of $P(x)$, and the quotient is $x^3 - 5x^2 + 4x + 1$.

C $x - 1$ is a factor of $P(x)$, and the quotient is $x^3 - 5x^2 + 6x + 1$.

D $x + 1$ is a factor of $P(x)$, and the quotient is $x^3 - 5x^2 + 6x - 1$.

E None of the above is correct.

Question 5

S is the complete set of values of x which satisfy both the inequalities:

$$-x^2 + 4x - 3 \geq 0 \quad \text{and} \quad x \geq 2$$

The set S can also be represented as a single inequality.

Which one of the following single inequalities represents the set S?

A $\dfrac{-x^2 + 4x - 3}{x - 2} \geq 0$

B $\dfrac{-x^2 + 4x - 3}{x - 2} \leq 0$

C $(-x^2 + 4x - 3)(x - 2) \geq 0$

D $(-x^2 + 4x - 3)(x - 2) \leq 0$

E $x^2 - 5x + 6 \geq 0$

F $x^2 - 5x + 6 \leq 0$

G $x \geq 2$

Question 6

Given that $\tan\theta = \dfrac{\sqrt{5}}{2}$ and $\theta \in \left(\pi, \dfrac{3\pi}{2}\right)$, and

$$M = \frac{1 + \sin\theta \times \cos\theta}{3\cos^2\theta - \sin^2\theta}$$

What is the value of M?

A $\dfrac{9+2\sqrt{5}}{7}$

B $\dfrac{9+2\sqrt{5}}{5}$

C $\dfrac{7+2\sqrt{5}}{5}$

D $\dfrac{5+2\sqrt{5}}{9}$

E None of the above is right.

Question 7

Consider the following two functions:

$$f(x) = |\log_{10} x|$$

For $g(x)$, it is an odd function and it has a period of 4, and:

$$g(x) = -x^2 + 2x, \quad 0 \leq x < 2$$

How many points of intersection do they have?

A 0

B 2

C 4

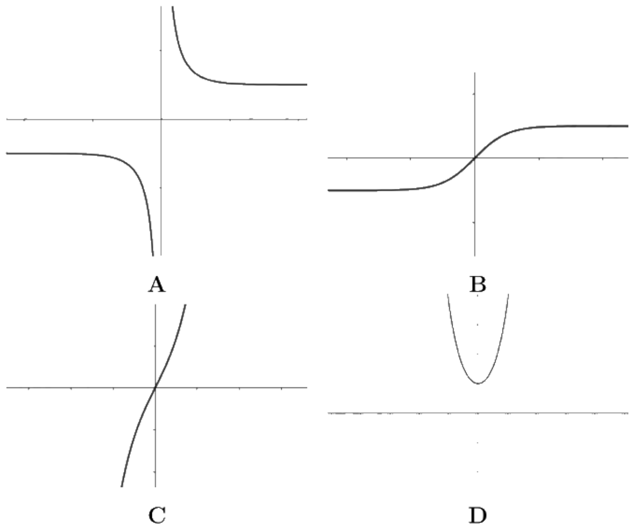
D 6

E 8

Question 8

Which one of the following shows the graph of

$$y = \frac{e^x + e^{-x}}{e^x - e^{-x}}\,?$$



A A

B B

C C

D D

Question 9

An arithmetic sequence has a first term 14.

Given that $S_5 > S_4$ and $S_5 > S_6$, what is one of the possible values of its common difference?

A -4

B -3

C -2

D $-\frac{14}{5}$

E $-\frac{7}{2}$

Question 10

Find the remainder when $2x^{2024} - 9x^{2022} - 81x^{2020} + 2x + 84$ is divided by $x^3 - 3x^2 - x + 3$.

Which of the following is the product of the coefficients?

A -280

B -290

C -300

D -320

E -330

Question 11

Given that

$$f(x) = \sqrt{x^4 - 3x^2 - 6x + 13} - \sqrt{x^4 - x^2 + 1}$$

Which of the following is the maximum of $f(x)$?

A $\sqrt{10}$

B $2\sqrt{3}$

C 4

D $2\sqrt{5}$

E $2\sqrt{6}$

Question 12

By using the trapezium rule with 2 strips to evaluate the integral of a parabola from 0 to 2, we need to calculate the area of an equilateral triangle.

Which of the following parabolas satisfies this condition?

A $y = -\sqrt{3}x^2 + 2\sqrt{3}x$

B $y = -\sqrt{3}x^2 + 4\sqrt{3}x$

C $y = \sqrt{3}x^2 - 6\sqrt{3}x$

D $y = x^2 + 4x$

E $y = x^2 - 2x$

Question 13

Find the total number of x, with x having the form of $\dfrac{\pi}{k}$, $k \in \mathbb{Z}$ and $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, for which

$$[(1 - \sin x)\cos x]' \geq 0$$

A 2

B 4

C 6

D 8

E 10

Question 14

Given the function

$$f(x) = x^3 - 3x^2 - 9x + 10$$

What is the value of $f(-2022) + f(-2020) + f(-2018) + ... + f(2020) + f(2022) + f(2024)$?

A -2022

B -2024

C 2024

D 2022

E 2020

Question 15

Given that for $f(x)$:

$$f(x+2)\,[1-f(x)] = 1+f(x)$$

Now, we know that $f(0)=2$, $f(1)=3$. What is the value of $f(2028)$?

A 0

B 2

C 3

D -2

E -3

F $-\frac{1}{2}$

G $-\frac{1}{3}$

Question 16

Given two functions:

$$f(x) = x^3 - 2x^2 - 3x + p$$

$$g(x) = \ln x$$

These two functions have 2 intersections that both have non-negative y-coordinates. Which of the following values of p can satisfy this condition?

A 0

B 1

C 2

D 3

E 4

Question 17

The two roots of the quadratic $12x^2 - 5x - 3$ are distinct real numbers. For every non-negative integer n, let s_n be the sum of the nth powers of these two roots.

What is the value of the infinite series

$$s_0 + s_1 + s_2 + ... ?$$

A $\frac{13}{9}$

B $\frac{17}{9}$

C $\frac{19}{4}$

D $\frac{17}{4}$

E $\frac{23}{4}$

Question 18

A line which passes (0, 0) is drawn tangent to the quadratic function $x^2 - 3x + b$ at the point on the curve where $x = 1$. Another line which passes (3, 0) is also drawn tangent to this curve.

What is the value of the total area of S, R and Q?

(Notice: P is not included.)

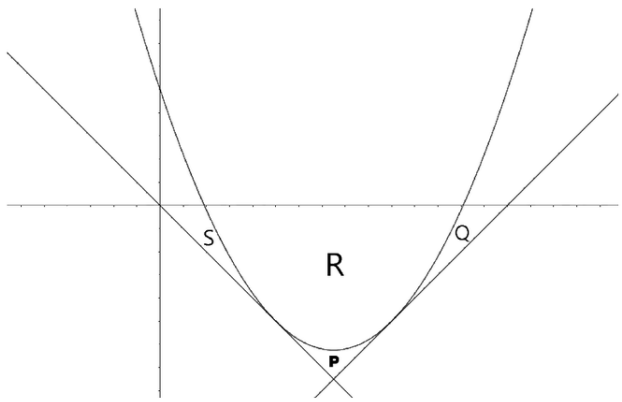


图 1. 19

A 2

B $\frac{13}{6}$

C $\frac{7}{3}$

D $\frac{15}{6}$

E $\frac{8}{3}$

Question 19

S_n is the sum of the first n terms of A_n. The first term $a_1 = 1$ and the sequence $\left\{\frac{S_n}{a_n}\right\}$ is an arithmetic sequence whose common difference is $\frac{1}{3}$.

Which of the following statements is true for all n?

A $\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n} < 2$

B $\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n} > 2$

C $\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n} < 1$

D $\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n} > 1$

E None of the statements above is true.

Question 20

If x is a real number, let $\lfloor x \rfloor$ be the greatest integer that is less than or equal to x. If n is a positive integer, let $S(n)$ be defined by

$$S(n) = \left\lfloor \frac{n}{10^{\lfloor \log n \rfloor}} \right\rfloor + 10\left(n - 10^{\lfloor \log n \rfloor} \times \left\lfloor \frac{n}{10^{\lfloor \log n \rfloor}} \right\rfloor\right)$$

(All the logarithms are base 10.)

How many integers n from 1 to 2011 (inclusive) satisfy $S(S(n)) = n$?

A 108

B 104

C 100

D 96

E 92