TMUA
COMMUNITY PAPERS

Qiuyun TMUA Simulation Test Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Qiuyun

Question 1

Denote the constant term of $\left[(p-2)x+\dfrac{p-1}{x^2}\right]^3$ as $f(p)$.

Find the sum of the x-coordinates of all the stationary points of $f(p)$.

A $\dfrac{4}{3}$

B 2

C $\dfrac{7}{3}$

D $\dfrac{10}{3}$

E $\dfrac{13}{3}$

Question 2

Given that $f(x) = \ln|x|$, find the value of

$$\int_{-1}^{1} \frac{\mathrm{e}^{x} + \mathrm{e}^{-x}}{\mathrm{e}^{x} - \mathrm{e}^{-x}} \left[f(-x) + f(x)\right] \,\mathrm{d}x$$

A e

B $-\mathrm{e}$

C 0

D $2\mathrm{e}$

E $-2\mathrm{e}$

Question 3

Which of the following functions provides a **counterexample** to the statement:

$f'(x) > 0$ is **necessary** for $f(x)$ being strictly monotonic.

A $f(x) = \ln x$, $x \in [0, +\infty]$

B $f(x) = x^5$, $x \in \mathbb{R}$

C $f(x) = \mathrm{e}^x$, $x \in \mathbb{R}$

D $f(x) = x^2$, $x \in \mathbb{R}$

E $f(x) = x$, $x \in \mathbb{R}$

Question 4

In a castle, there are five knights named Elvin, Black, Carl, Daniel and Ethan. Each of them has a statement about their own skill level on their helmet. While each knight's actual swordsmanship skill level is the same, their statements are all different:

Elvin: My swordsmanship level is an even number greater than 2.

Black: My swordsmanship level is neither 1 nor 4.

Carl: My swordsmanship level is not a prime.

Daniel: My swordsmanship level is neither 2 nor 5.

Ethan: My swordsmanship level is an odd number.

Similarly, only one knight is telling the truth.

Which knight's statement is true?

A Elvin

B Black

C Carl

D Daniel

E Ethan

Question 5

What is the area of the region enclosed by the graph of the equation

$$x^2 + y^2 = |x| + y\,?$$

A $1 + \dfrac{3\pi}{4}$

B $\dfrac{1}{2} + \dfrac{3\pi}{4}$

C $1 + \dfrac{\pi}{4}$

D $\dfrac{1}{2} + \dfrac{\pi}{4}$

E $\dfrac{3}{2} + \dfrac{\pi}{4}$

Question 6

If one of the expansions of $(2x^3 + y^2)^n$ is tx^6y^6, what is the area between the curve $y = \dfrac{9}{40}tx$ and the curve $y = \dfrac{3}{20}tx^2$?

A 1

B 3

C $\dfrac{27}{8}$

D $\dfrac{15}{4}$

E $\dfrac{33}{8}$

Question 7

What is the value of

$$\log_3 7 \times \log_5 9 \times \log_7 11 \times \log_9 13 \times \cdots \times \log_{21} 25 \times \log_{23} 27\,?$$

A 2

B 4

C 6

D 8

E 10

Question 8

For the sequence $\{a_n\}$ $(a_n \in \mathbb{N}^+)$, we define b_k as the maximum in $a_1, a_2, \dots, a_k$ $(k = 1, 2, \dots n)$ $(n \in \mathbb{N}^+)$. We call the sequence $\{b_n\}$ the "M sequence" of $\{a_n\}$.

Which of the following statements are **necessarily** true?

I. If $\{a_n\}$ is a monotonically decreasing sequence, then $\{b_n\}$ is a constant sequence.

II. If $\{a_n\}$ is a monotonically increasing sequence, then $b_n = a_n$.

III. There are 8 forms of $\{a_n\}$ that can satisfy $\{b_n\} = 2, 3, 3, 5, 5$.

A I

B II

C III

D I and II

E I and III

F I and II and III are right.

Question 9

The set of values of x that satisfies $ax^2 + bx + c > 0$ is $x < -5$ or $x > 6$.

Which of the following statements **must** be true?

I. $a > 0$

II. $a + b + c > 0$

III. The set of values of x that satisfies $cx^2 - bx + a > 0$ is $x < -\frac{1}{6}$ or $x > \frac{1}{5}$.

A Only I is right.

B Only II is right.

C Only III is right.

D I and II are right.

E I and III are right.

F I and II and III are right.

Question 10

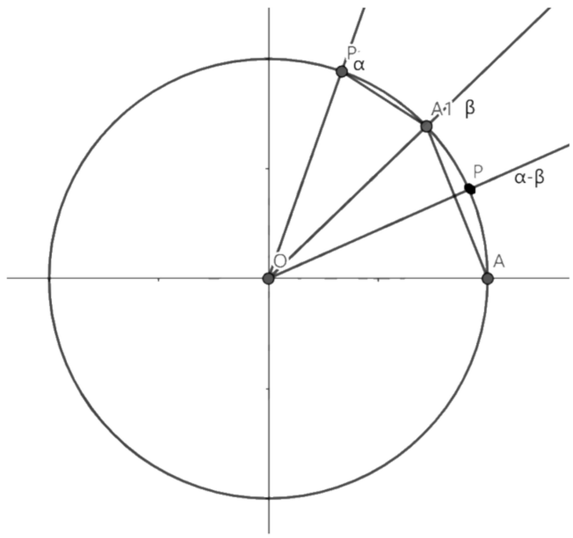
As is shown in the graph, the unit circle intersects with the x-axis at the point $A(1,\, 0)$. The terminal sides of the acute angles α, β, $\alpha - \beta$ are formed by starting from $O(0,\, 0)$ and their initial sides along the positive x-axis.

Which of the following statements **must** be true?

I. The area of the sector A_1OP_1 is $\dfrac{1}{2}(\alpha - \beta)$.

II. If A_1 and P overlap, $|AP_1| = 2\sin\beta$.

III. If $\alpha = \dfrac{\pi}{3}$, the area of the quadrilateral AA_1P_1O ranges from $\dfrac{\sqrt{3}}{4}$ to 1.



A Only I is right.

B Only II is right.

C Only III is right.

D I and II are right.

E I and III are right.

F I and II and III are right.

Question 11

Your classmate Max mentioned Mersenne primes and then said that numbers of the form $2^n - 1$ are always prime.

Which of the following is a **counterexample** to Max's statement?

A 7

B 15

C 24

D 31

E 65

Question 12

Now we need to prove:

$$\log_a b = \frac{\ln b}{\ln a}$$

How can we choose these steps and prove this statement?

(1) Assume $\ln b = m$ and $\ln a = n$

(2) $\mathrm{e}^m = b$, $\mathrm{e}^n = a$

(3) $b^m = \mathrm{e}$, $a^n = \mathrm{e}$

(4) $\log_a b = mn$

(5) $\log_a b = \dfrac{m}{n}$

(6) $\log_a b = \dfrac{n}{m}$

(7) $a^{\frac{m}{n}} = b$

(8) $\log_a b = \log_{\mathrm{e}^n} \mathrm{e}^m$

(9) $a^{mn} = b$

(10) $\log_a b = \dfrac{\ln b}{\ln a}$

A (1), so (2), so (7), so (5), so (10).

B (1), so (2), so (8), so (4), so (10).

C (1), so (2), so (7), so (4), so (10).

D (1), so (2), so (8), so (5), so (10).

E (1), so (3), so (7), so (4), so (10).

F (1), so (3), so (8), so (5), so (10).

Question 13

Use the trapezium rule to approximate the value of the following definite integral, with the number of intervals $n = 4$:

$$S = \int_{\pi}^{5\pi} (\sin x + x)\,\mathrm{d}x$$

The following lines are the attempt to solve this integral. Which of the following lines is wrong?

I. The step size h is $\frac{5\pi - \pi}{4} = \pi$.

II. Then, we can get the intermediate points and the boundary points: $x_0 = \pi$, $x_1 = 2\pi$, $x_2 = 3\pi$, $x_3 = 4\pi$, $x_4 = 5\pi$.

III. By applying the trapezium rule: $S = \frac{1}{2}\left(f(\pi) + f(2\pi) + f(3\pi) + f(4\pi) + f(5\pi)\right)$

IV. Finally, we get the exact solution: $S = \frac{1}{2}(\pi + 2\pi + 3\pi + 4\pi + 5\pi) = \frac{15}{2}\pi$

A Line I.

B Line II.

C Line III.

D Line IV.

E The solution is correct.

Question 14

Here is a true statement:

A function $f : M \to M$ is said to be a **contraction mapping** if there exists a number $C < 1$ such that $|f(x) - f(y)| \leq C \times |x - y|$ for all $x, y \in M$.

Which of the following statements **must** be true?

A If a function $f : M \to M$ is not a contraction mapping, then for any $C \geq 1$, there exist some $x, y \in M$ such that $|f(x) - f(y)| > C \times |x - y|$.

B If a function $f : M \to M$ is not a contraction mapping, then for any $C \geq 1$, we have $|f(x) - f(y)| \leq C \times |x - y|$ for all $x, y \in M$.

C If a function $f : M \to M$ is not a contraction mapping, then for any $C < 1$, there exist some $x, y \in M$ such that $|f(x) - f(y)| > C \times |x - y|$.

D If a function $f : M \to M$ is not a contraction mapping, then for any $C < 1$, we have $|f(x) - f(y)| \leq C \times |x - y|$ for all $x, y \in M$.

E If there exists a number $C \geq 1$ such that $|f(x) - f(y)| > C \times |x - y|$ for all $x, y \in M$, then this function $f : M \to M$ is not a contraction mapping.

F If there exists a number $C \geq 1$ such that $|f(x) - f(y)| \leq C \times |x - y|$ for all $x, y \in M$, then this function $f : M \to M$ is not a contraction mapping.

Question 15

You and your friend are playing a game of distributing weights. Each of you has a bag containing a certain real, discrete amount of weight (unit: kg). Let x represent the weight in your bag, and y represent the weight in your friend's bag. You want to adjust the weights in your bags such that $x - y$ is minimised, while satisfying the following conditions:

1. Your friend must have at least 1 kg in their bag.

2. The weights of your friend cannot exceed twice the weights of yours minus 1.

3. The total weight in both bags cannot exceed an unknown value m.

After calculating, you find that the minimum value of the difference between your weight and your friend's is -1, i.e. $x - y = -1$. Now, you need to find the maximum allowed total value m that satisfies these conditions.

A 2

B 3

C 4

D 5

E 6

F 7

Question 16

Given two functions:

$$f(x) = \left|x^2 - 6x + 1\right|$$

$$g(x) = 4x + \frac{4}{x}$$

Now, $m \in \mathbb{R}$ is a constant and denote $\epsilon = f(x) - g(m)$. The function about ϵ is:

$$\epsilon^2 + 8\epsilon = 0$$

Find the maximum number of the solutions of x.

A 1

B 2

C 3

D 4

E 5

F 6

G 7

H 8

Question 17

Consider the following attempt to solve the integral:

$$R = \int_0^{\pi} \mathrm{e}^{\cos x} \times \left(\sin^3 x\right) \, \mathrm{d}x$$

I. $u = \cos x, \ \dfrac{\mathrm{d}u}{\mathrm{d}x} = -\sin x, \ x = 0 \Rightarrow u = 1, \ x = \pi \Rightarrow u = -1$

II. $R = \displaystyle\int_1^{-1} \mathrm{e}^u \times \sin^3 x \times \frac{1}{-\sin x} \, \mathrm{d}u$

III. $R = \displaystyle\int_1^{-1} -\mathrm{e}^u (1 - u^2) \, \mathrm{d}u$

IV. $R = \displaystyle\int_{-1}^{1} \mathrm{e}^u \, \mathrm{d}u - \int_{-1}^{1} \mathrm{e}^u u^2 \, \mathrm{d}u$

V. $= \left[\mathrm{e}^u\right]_{-1}^{1} - \left[(u^2 - 2u + 2)\mathrm{e}^u\right]_{-1}^{1}$

VI. $= \mathrm{e} - \dfrac{1}{\mathrm{e}} - \left(\mathrm{e} - \dfrac{1}{\mathrm{e}} - \left(2\mathrm{e} - \dfrac{2}{\mathrm{e}}\right) + \left(2\mathrm{e} - \dfrac{2}{\mathrm{e}}\right)\right) = 0$

Which one of the following is true?

- **A** The solution is correct.
- **B** The solution is wrong; the error arises as a result of (I).
- **C** The solution is wrong; the error arises as a result of (II).
- **D** The solution is wrong; the error arises as a result of (III).
- **E** The solution is wrong; the error arises as a result of (IV).
- **F** The solution is wrong; the error arises as a result of (V).
- **G** The solution is wrong; the error arises as a result of (VI).

Question 18

The original function $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ is rotated by $\dfrac{\pi}{2}$ anticlockwise around the origin $(0, 0)$.

Which of the following is the final function?

A $\ln \dfrac{x-1}{x+1}$

B $\ln \sqrt{\dfrac{x-1}{x+1}}$

C $\ln \dfrac{x+1}{x-1}$

D $\ln \sqrt{\dfrac{x+1}{x-1}}$

E None of the above is correct.

Question 19

$f(x)$ is continuous and now given the proposition

$$Z : \int_0^2 f(x)\,\mathrm{d}x = 0 \quad \text{and} \quad f(1) = 0$$

which of the following options correctly describes the relationship between the given proposition and the original proposition?

M: $\displaystyle\int_0^1 f(x)\,\mathrm{d}x = -\int_1^2 f(x)\,\mathrm{d}x$

N: $f(x) + f(2-x) = 0$

P: $\displaystyle\int_0^1 f(x)\,\mathrm{d}x = 0$ and f has a period of 1.

A M is necessary and sufficient for Z; N is necessary and sufficient for Z; P is necessary and sufficient for Z.

B M is necessary and sufficient for Z; N is only sufficient for Z; P is only necessary for Z.

C M is only necessary for Z; N is necessary and sufficient for Z; P is only necessary for Z.

D M is only necessary for Z; N is only sufficient for Z; P is neither necessary nor sufficient for Z.

E M is necessary and sufficient for Z; N is only sufficient for Z; P is neither necessary nor sufficient for Z.

Question 20

Point $A(1,\ 1)$ is on the parabola C: $x^2 = 2py$ $(p > 0)$. Given the statement

S: A line with gradient k passing through $B(0,\ -1)$ intersects the parabola at two points P and Q.

Which of the following statements is **necessary** for S?

A $k < 0$

B $k > 0$

C $k < -1$ or $k > 1$

D $k < -2$ or $k > 2$

E $k < 0$ or $k > 0$