TMUA

COMMUNITY PAPERS

Rosie's Papers Set A Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Rosie

Question 1

A geometric series with n terms is denoted by S, where $S = i + i^2 + i^3 + \cdots i^n$, and $i = \sqrt{-1}$. Find an expression for the sum of all the **unique** possible values of S

[Hint: A denominator with i can be 'rationalised']

A 0

B $2i$

C $-1 + i$

D $-2 + 2i$

E $-\frac{1}{2} + \frac{1}{2}i$

Question 2

$\lfloor x \rfloor$ is defined as the greatest integer **less than or equal** to x, e.g. $\lfloor 6.7 \rfloor = 6$, $\lfloor 5 \rfloor = 5$, $\lfloor \pi \rfloor = 3$. Hence find the number of real solutions to the below equation

$$\lfloor x \rfloor^2 - 3\lfloor x \rfloor - 2x + 5 = 0$$

A 0

B 1

C 2

D 3

E 4

F 5

G 6

H An infinite amount

Question 3

A group of 6 students, 4 girls and 2 boys, are sat around a circular table discussing how much they hate Stats. How many ways can the students arrange themselves, so no boys are sitting next to each other? (Note: Two arrangements are considered the same if they can be arranged into one another)

A 72

B 144

C 240

D 480

E 600

Question 4

Find the product of all the real roots of the equation $x^4 - 4x^2 + 2 = 0$

A -2

B 2

C 0

D 4

E 1

Question 5

Given that function f satisfies $3f(x) + 2f\left(\frac{x+2}{x-1}\right) = x + 2$, $x \in \mathbb{R}$ and $x \neq 1$. Find the value of $f(4)$.

A $\frac{4}{5}$

B 0

C $\frac{14}{5}$

D 2

E 4

Question 6

Find the simplified value given by the below sum, where x is in degrees.

$$\sum_{x=1}^{90} \cos 4x$$

A -1

B 45

C $\frac{1}{2}$

D 1

E 0

F $-\frac{1}{2}$

Question 7

Two circles, C_1 and C_2, are defined below:

$$C_1 : x^2 + y^2 = 8$$

$$C_2 : x^2 + y^2 - 6x + 8y = -22$$

Find the shortest distance between them.

A 0

B $5 - \sqrt{11}$

C $5 - 2\sqrt{2} + \sqrt{3}$

D $5 - 2\sqrt{2} - \sqrt{3}$

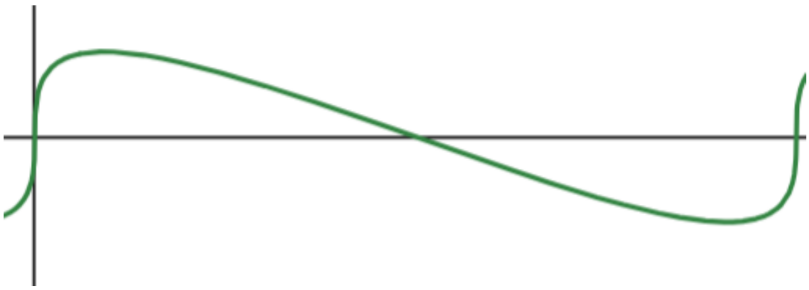
E $5 + 2\sqrt{3} - \sqrt{2}$

F $5 - \sqrt{5}$

G 3

Question 8

The curve below is shown roughly between $0 \leq x \leq 2\pi$. The curve passes through $(0, 0)$, $(\pi, 0)$ and $(2\pi, 0)$. It is also strictly bounded between $y = -1$, $y = 1$ and its maximum value in the range $0 \leq x \leq \pi$ occurs at a point where $x < \frac{\pi}{2}$. Find its equation from the list below.



A $\sin(x) = y^3$

B $\sin(x - y) = y$

C $\sin(x + y) = y$

D $\sin(xy) = y$

E $\cos(x + y) = y$

F $\cos(x) = y^3$

G $\cos(x - y) = x$

H $\cos(y) = x$

Question 9

$f(x)$ is given by $f(x) = ax^b - cx^d$, where b and d are positive integers with $b < d$, and $f(x)$ is also given to have these below properties:

$$\int_0^1 f(x)\,\mathrm{d}x = \frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 0$$

$$f(1) = 1$$

Hence find $a + b + c + d$

A 6

B 8

C 10

D 12

E it cannot be determined uniquely without an initial boundary condition for $f(0)$

Question 10

A function $f(x)$ is defined by $f(x) = x^2 - 4x$. A real constant c satisfies the equation

$$\int_0^c f(x)\,\mathrm{d}x = \int_c^4 f(x)\,\mathrm{d}x$$

Hence find the value of

$$\int_0^2 f(x+c)\,\mathrm{d}x$$

A $-\frac{22}{3}$

B $-\frac{16}{3}$

C $-\frac{8}{3}$

D 0

E $\frac{16}{3}$

F $-\frac{32}{3}$

Question 11

It is given that the expression $\frac{3}{\sqrt[3]{4}-1}$ can be rewritten as $\sqrt[3]{a}+\sqrt[3]{b}+1$

Hence find the sum of a and b

A 20

B 16

C 4

D 0

E 67

F 3

Question 12

How many solutions does $(\sin x + \cos x)\cos^2 x = 0$ have in the domain $0 \leq x \leq \pi$

A 0

B 1

C 2

D 4

E Infinite

Question 13

Find the coefficient of xy^2z^3 in the expansion of $(7x + 2y + 3z)^6$

A 90720

B 2520

C 756

D 0

E 45360

Question 14

A student taking the S.T.E.P. maths exam asks n of their friends also sitting the exam what they think they'll get on the first paper and takes the average, finding it to be 64. When the student adds their own predicted score to the sample the mean increase by 2 marks. An international friend then contacts this student to add their own predicted score, 90 marks, with the final average coming to 68 marks as a result.

Hence find n

A 10

B 15

C 12

D 5

E 8

Question 15

A line with equation $y = mx + c$, where $m > 0$, and a curve with equation $y = x^2 + 2$ both lie on the xy plane, with the line being a tangent to the curve and intercepting the x-axis at $x = -\frac{1}{2}$. Find the area between the line, the curve and the y-axis

A $\frac{4}{3}$

B $\frac{2}{3}$

C 1

D $\frac{1}{3}$

E $\frac{6}{7}$

F Undefined

Question 16

A recurrence relation is given by $x_{n+1} = \frac{x_n^2+1}{2}$, where $x_1 = 0.75$

Choose the closest estimate to x_{1000}

A 0

B 0.5

C 0.85

D Infinity

E 1

Question 17

A student named James Chen is approximating the value of the below integral using the Trapezium rule with n strips.

$$\int_0^2 x^3 - 3x^2 \, \mathrm{d}x$$

The approximation is given by T and the true value is given by I, hence which of the following statements is correct?

A $T < I$ for all integers $n \geq 1$

B $T > I$ for all integers $n \geq 1$

C There exists a finite integer k such that $T < I$ and $n < k$, and $T > I$ when $n \geq k$

D There exists a finite integer k such that $T > I$ and $n < k$, and $T < I$ when $n \geq k$

E Whether $T > I$ or $T < I$ depends on the specific value of n, alternating as n increases

F $T = I$ for all integers $n \geq 1$

Question 18

A circle C has a radius of r and is centred at the origin $(0, 0)$. A fixed point P lies outside the circle at the coordinates $(d, 0)$, where $d > r$. Two tangent lines are drawn from P to touch the circle at two points, A and B. A chord is then drawn connecting A and B, with this chord intersecting the x-axis at a point Q.

Which of the following expressions represents the x-coord of Q

A $\frac{r^2}{d}$

B $\frac{r^3}{d^2}$

C $d - \frac{r^2}{d}$

D $\sqrt{d^2 - r^2}$

E $\frac{rd}{\sqrt{d^2 - r^2}}$

Question 19

A polynomial $P(x)$ leaves a remainder of 3 when divided by $(x-1)$, and a remainder of -1 when divided by $(x+3)$

Let $R(x)$ be the remainder when $P(x)$ is divided by the quadratic expression $(x-1)(x+3)$

Which of the following expressions is equal to $R(x)$

A $2x+1$

B $x+2$

C $x-2$

D $-x+4$

E $2x-1$

Question 20

An arithmetic progression has a non-zero common difference d.

The 1st term, 4th term and the 10th term of this arithmetic progression form the first three consecutive terms of a geometric progression, **in that specific order**.

Which of the following statements about the arithmetic progression must be true

A The first term is equal to the common difference

B The first term is three times the common difference

C The first term is half of the common difference

D The sum of the first 10 terms of the arithmetic progression is equal to zero

E The 10th term of the arithmetic progression is equal to zero