TMUA

COMMUNITY PAPERS

StepMaths Mock Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: StepMaths

Question 1

Given that the coefficient of x^2 in the expansion of $(1+x^2)(3-2x)^n$ is 1323, is n even or odd?

A Odd

B Even

C Either, but more odd solutions

D Either, but more even solutions

E There are no solutions

Question 2

The polynomial $P(x)$ has $(x-1)^2$ as a factor. Given that $P(x)$ is of the form $P(x) = 2x^4 - 3x^3 + ax^2 + bx - 6$, find a and b.

A $a = -4,\ b = 11$

B $a = -4,\ b = -11$

C $a = -6,\ b = 13$

D $a = -6,\ b = -13$

E There is insufficient information to determine a and b

Question 3

What is the maximum value of $\sqrt{x^2+y^2}$ for x and y that satisfy the inequality $x^2-30x+y^2-16y \leq -253$?

A 6

B 11

C 17

D 23

E There is no maximum value

Question 4

The function $f(x)$ satisfies the following equation for $x \neq 1$:

$$f(x) + xf(1-x) = \frac{x}{x-1}$$

Given that $f(2)$ can be written as a fraction $\frac{a}{b}$ in its lowest terms, find the value of $a + b$.

A -2

B -1

C 2

D 4

E 5

Question 5

$f(x) = x^2 + \frac{7}{2}$. What is the area enclosed by $f(x)$ and $f^{-1}(x)$?

A $\frac{1}{24}$

B $\frac{5}{48}$

C $\frac{7}{48}$

D $\frac{5}{24}$

E $\frac{7}{24}$

Question 6

The ratio of the 11^{th} and the 21^{st} terms of an arithmetic progression is $9 : 8$. What is the ratio of the 51^{st} and 91^{st} terms of the arithmetic progression?

A $1 : 5$

B $5 : 1$

C $10 : 11$

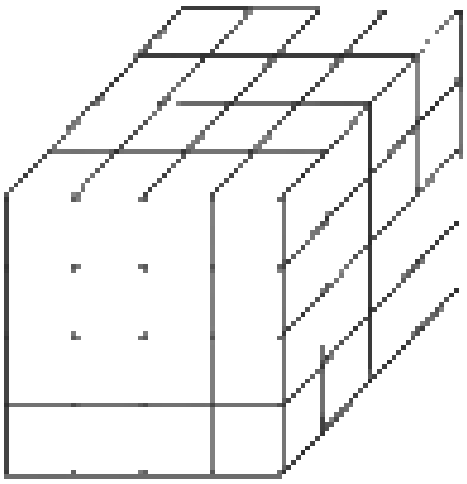
D $11 : 10$

E $1 : 1$

Question 7

A white wooden cube is made of n^3 identical, smaller cubes, where n is an even number. Each small cube on the surface of the large cube is numbered with a "0". The numbered cubes are then removed, and the small cubes on the new surface of the remaining cube are all numbered with "1". The process repeats, so that we have small cubes with numbers $0, 1, 2, 3, ...$

The wooden block is now dismantled into the n^3 constituent cubes and put into a bag. Small cubes are picked consecutively at random without replacement. As n increases, which of the following is true of the probability that all the numbers from 0 to $\frac{n}{2}$ inclusive have been picked before any number repeats itself?



A It increases towards 1

B It decreases towards 0

C It increases towards a limit less than 1

D It decreases towards a limit greater than 0

E It is constant

Question 8

How many solutions are there to the equation $\sin(x) + \cos(x) = 1$ for $-2\pi \leq x \leq 2\pi$?

A 0

B 4

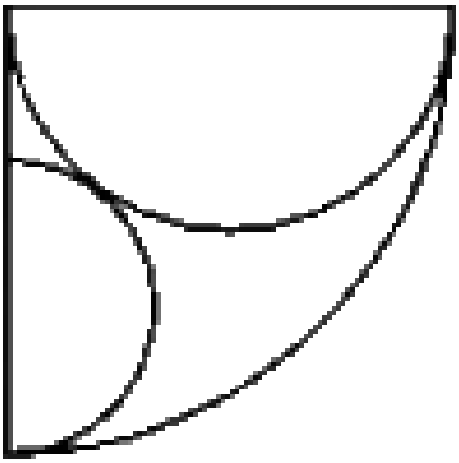
C 5

D 8

E 9

Question 9

In the diagram, the radius of the large quadrant is 2. The two semicircles touch each other only once. Find the area inside the quadrant but outside the two semicircles.



A $\frac{5\pi}{36}$

B $\frac{5\pi}{18}$

C $\frac{5\pi}{9}$

D $\frac{7\pi}{9}$

E $\frac{\pi}{2}$

Question 10

The equation $\sin(x) = kx$ has 5 solutions, where $k > 0$. What is the simplest complete set of conditions that k satisfies when this is true? (You may use the fact that $\frac{d}{dx}\sin x = \cos x$.)

A $2\pi < \frac{\sqrt{1-k^2}}{k} < \frac{5\pi}{2}$ and $\tan(\arccos(k)) = \arccos(k)$

B $2\pi < \frac{\sqrt{1-k^2}}{k} < \frac{5\pi}{2}$ and $\tan\left(\frac{\sqrt{1-k^2}}{k}\right) = \frac{\sqrt{1-k^2}}{k}$

C $2\pi < \frac{\sqrt{1-k^2}}{k} < \frac{5\pi}{2}$

D $\tan(\arccos(k)) = \arccos(k)$

E $\tan\left(\frac{\sqrt{1-k^2}}{k}\right) = \frac{\sqrt{1-k^2}}{k}$

Question 11

Let the solutions to the equation $2^{2x+1} - 2^x - 2^{x+3} + 2^2 = 0$ be a and b. Find the value of $a + b$.

A 1

B -1

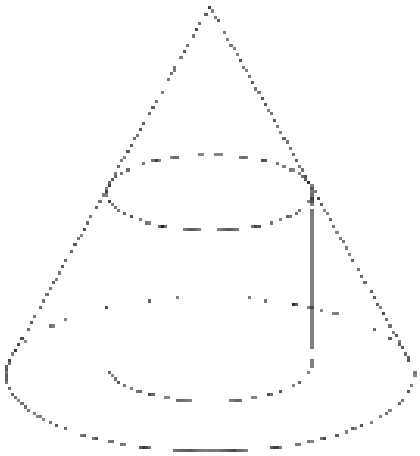
C $\frac{5}{2}$

D $-\frac{5}{2}$

E 4

Question 12

A right circular cone has base radius R and height H, where $R < H$. A cylinder is inscribed in the cone in such a way that the curved surface of the cone is tangential to the cylinder as shown in the diagram below. Find the maximum volume of the cylinder in terms of R and H only.



A $\frac{1}{3}\pi R^2 H$

B $\frac{\pi H^2 R}{2(H-R)}$

C $\frac{2R}{3}$

D $\frac{4\pi R^2 H}{9}$

E $\frac{4\pi R^2 H}{27}$

Question 13

How many roots does the equation $3\cos^4(x) + 2\cos^3(x) - 7\cos(x) + 1 = 0$ have in the range $0 < x < 2\pi$?

A 0

B 1

C 2

D 4

E 8

Question 14

Let k_t be a sequence of numbers defined by $k_1 = 1$ and satisfying

$$\frac{k_{t+1}}{k_t} = 2^t$$

Find $\log_2(k_{100})$.

A $\log_2 5050$

B 4950

C 5050

D 2^{4950}

E 2^{5050}

Question 15

At the two points where the line $y = k$ cuts the curve $y = x^2$, tangents are drawn. It is found that they intersect at the point $(0, -2)$. What is the value of k?

A 2

B $\sqrt{2}$

C -2

D $-\sqrt{2}$

E There is no such k

Question 16

For what values of x can the following simultaneous equations be solved?

$$4^{\frac{x}{y}+\frac{y}{x}} = 32$$

$$\log_3(x-y) + \log_3(x+y) = 1$$

A There are no solutions

B $-2 \leq x \leq 2$

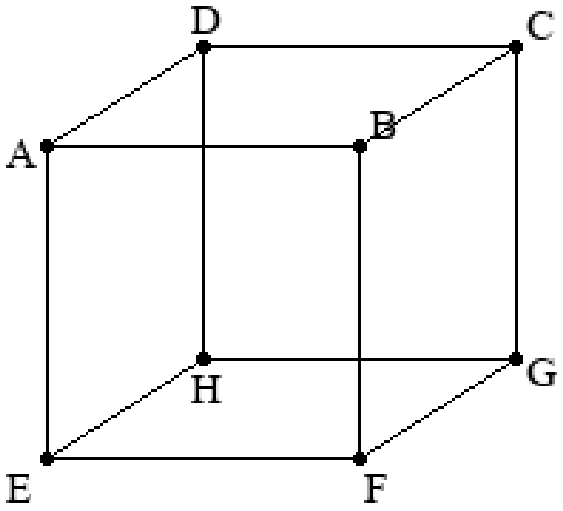
C $x = 2$ or $x = -2$

D $x = -2$ only

E $x = 2$ only

Question 17

A cube has each side length 2 units. An ant walks on the surface of the cube and starts at H. What is the shortest distance it has to travel to reach the midpoint of BF?



A $\sqrt{13}$

B $1 + 2\sqrt{2}$

C 5

D $\sqrt{29}$

E $3 + 2\sqrt{2}$

Question 18

If $x + \frac{1}{x} = 5$ what is the value of $x^4 + \frac{1}{x^4}$?

A 723

B 625

C 527

D 525

E 519

Question 19

What is the total length of the interval or intervals on which the following inequality is true?

$$-1 < \frac{3x+4}{x-6} < 1$$

A 0

B $\frac{1}{2}$

C $\frac{9}{2}$

D $\frac{11}{2}$

E Infinite

Question 20

How many positive integers are there such that x and $x + 99$ are both square numbers?

A 0

B 1

C 3

D 6

E Infinitely many