TMUA

COMMUNITY PAPERS

# StepMaths Mock Paper 2

**Recommended time:** 75 minutes

**Calculators:** not permitted

**Format:** 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

**Paper guidance**

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

**Source credit:** StepMaths

## Question 1

A letter L is printed somewhere on the $x$-$y$ plane. It then undergoes three transformations: it is rotated anti-clockwise about the origin by $\frac{\pi}{4}$ radians, then it is reflected in the $x$-axis, and finally it is rotated clockwise about the origin by $\frac{\pi}{4}$ radians. Which of the following transformations should be used to undo the change to the L?

**A** Rotate clockwise about the origin by $\frac{\pi}{2}$ radians

**B** Rotate clockwise about the origin by $\frac{\pi}{4}$ radians, then reflect in the $x$-axis, then rotate anti-clockwise about the origin by $\frac{\pi}{4}$ radians

**C** Reflect in the line $y = x$

**D** Reflect in the line $y = -x$

**E** Reflect in the $x$-axis

## Question 2

What is the value of

$$\int_{-a}^{a} \sqrt{x^4 + \frac{a^4}{x^4} + 2a^2}\, dx$$

**A** $\frac{2}{3}(a^3 + 3a)$

**B** $\frac{2}{3}(a^3 - 3a)$

**C** $\frac{2}{3}\left(a^3 + \frac{1}{a}\right)$

**D** $\frac{2}{3}\left(a^3 - \frac{1}{a}\right)$

**E** Its value is not defined

## Question 3

How many solutions are there to the equation $\cos^3(x) - \frac{5\cos^2(x)}{2} + 2\cos(x) - \frac{1}{2} = 0$ for $0 < x < 2\pi$?

**A** 0

**B** 2

**C** 3

**D** 4

**E** 6

## Question 4

In Mauritius, the Prime Minister only tells the truth on Mondays, Tuesdays, Wednesdays and Thursdays. He tells lies on all other days. The President only tells the truth on Fridays, Saturdays, Sundays and Mondays. He tells lies on all other days. On one day last week they both said: "I lied yesterday!". On which day of the week was that?

**A** Monday

**B** Thursday

**C** Friday

**D** There is more than one possibility

**E** The situation is impossible

## Question 5

Claim: For any real function $f$ and real number $x$, if $f(f(x)) = x$, then $f(x) = f^{-1}(x)$.

Which of the following is a counter example to the above claim?

**A** $f(x) = x \ \forall x \in \mathbb{R}, \quad x = -1$

**B** $f(x) = \frac{1}{x} \ \forall x > 0, \quad x = 1$

**C** $f(x) = \frac{1}{x} \ \forall x > 0, \quad x = -1$

**D** $f(x) = 1 \ \forall x \in \mathbb{R}, \quad x = -1$

**E** $f(x) = 1 \ \forall x \in \mathbb{R}, \quad x = 1$

## Question 6

Two sequences are related by the following relationships:

$$u_{n+1} = xu_n, \quad u_1 = 1 \qquad q_n = \frac{d(u_n)}{dx}$$

Find the value of

$$\int_0^x \sum_{n=1}^{\infty} q_n \, dx$$

if $|x| < 1$.

**A** $x$

**B** $\frac{x}{1-x}$

**C** $\frac{1}{1-x}$

**D** The integral is not defined

**E** We cannot tell if the integral is defined

## Question 7

Use some or all of the lines below to prove the inequality

$$(a+b)\left(\frac{1}{a}+\frac{4}{b}\right)\geq 9$$

holds if $a>0$ and $b>0$.

1. $(a+b)\left(\frac{1}{a}+\frac{4}{b}\right)\geq 9 \iff \frac{b}{a}+\frac{4a}{b}+5\geq 9$ 2. $\frac{b}{a}+\frac{4a}{b}+5\geq 9 \iff \frac{b}{a}+\frac{4a}{b}-4\geq 0$ 3. $\frac{b}{a}+\frac{4a}{b}-4\geq 0 \iff \frac{4a^2-4ab+b^2}{ab}\geq 0$ 4. $\frac{b}{a}+\frac{4a}{b}-4\geq 0 \iff \frac{4a^2-4ab+b^2}{ab}\geq 0$ if and only if $ab>0$ 5. $\frac{4a^2-4ab+b^2}{ab}\geq 0 \iff \frac{(2a-b)^2}{ab}\geq 0$ 6. We have $\frac{(2a-b)^2}{ab}\geq 0$ if and only if $a>0$ and $b>0$ 7. We have $\frac{(2a-b)^2}{ab}\geq 0$ if $a>0$ and $b>0$ 8. Therefore, $(a+b)\left(\frac{1}{a}+\frac{4}{b}\right)\geq 9$ if $a>0$ and $b>0$ 9. Therefore, since $a>0$ and $b>0$, $(a+b)\left(\frac{1}{a}+\frac{4}{b}\right)\geq 9$

**A** $1,2,3,5,7,8$

**B** $1,2,4,5,6,8$

**C** $1,2,4,5,6,9$

**D** $1,2,3,5,7,9$

**E** $1,2,3,5,6,9$

## Question 8

3 consecutive positive integers are chosen randomly. Which of the following statements **must** be true?

1. 1 of them is divisible by 3 2. At least 1 of them is divisible by 3 3. The product of the 3 numbers is divisible by 6 4. The sum of the 3 consecutive numbers is divisible by 3 5. The sum of the 3 consecutive numbers is odd

**A** $1, 3, 4$

**B** $1, 2, 3, 4, 5$

**C** $2, 3, 4$

**D** $1, 2, 3, 4$

**E** $2, 3, 4, 5$

## Question 9

A triangle is inscribed in a circle so that its vertices lie on the circumference. How many of the following statements are individually sufficient to imply that it is an acute-angled triangle (i.e. all of its angles are strictly between 0 and $\frac{\pi}{2}$ radians)?

1. None of its sides is a diameter of the circle 2. Every diameter of the circle meets the triangle 3. If the triangle is reflected in its longest edge, the reflected triangle meets the circle 4. If the triangle is reflected in its longest edge, the reflected triangle meets the circle at least three times 5. Its lengths and angles satisfy the cosine rule 6. The centre of the triangle is the centre of the circle.

**A** 1

**B** 2

**C** 3

**D** 4

**E** 5

## Question 10

Which of the following is sufficient to imply that an integer $x$ is not a square number?

1. $|x|$ has an even number of factors 2. The last digit of $x$ is a 9 and the first digit of $x$ is a 2. 3. $x$ is not positive

**A** All of them

**B** 1 and 3

**C** 2 and 3

**D** 1 only

**E** None of them

## Question 11

A function $f(x)$ of real numbers is called even if and only if $f(x) = f(-x)$ for all $x$ in the domain of $f(x)$. It is called odd if and only if $f(x) = -f(-x)$ for all $x$ in the domain.

For instance:

- $f(x) = |x|$ for all $x \in \mathbb{R}$ is even
- $f(x) = x$ for all $x \in \mathbb{R}$ is odd
- $f(x) = \sqrt{x^2 - 4}$ for $x$ with $|x| \geq 2$ is even
- $f(x) = \sqrt{x - 2}$ for $x$ with $x \geq 2$ is neither even nor odd

Which of the following statements **must** be true?

1. If $f(x)$ is odd, then $\frac{f'(x)}{f'(-x)} = -1$ for all $x$ in the domain 2. If $f(x)$ is even, then $\frac{f'(x)}{f'(-x)} = -1$ for all $x$ in the domain 3. If $f(x)$ is odd, then $f(0) = 0$ 4. If $f(x)$ is even, then $f(0) = 0$ 5. If $f(x)$ is odd, then $\int_{-a}^{a} f(x)dx = 0$ for all $a$ in the domain 6. If $f(x)$ is even, then $\int_{-a}^{a} f(x)dx = 0$ for all $a$ in the domain

**A** $1, 2, 3, 5$

**B** $2, 3, 4, 6$

**C** $2, 5$

**D** Only 5

**E** None of them

## Question 12

Integers $x$ and $y$ satisfy $2x^6 + y^2 - 2x^3y = 80$. Which of the following can be deduced?

1. $x = 2$ 2. $x^6 < 80$ 3. $y > 0$ 4. $y$ is even

**A** All of them

**B** $1, 2, 4$

**C** $2, 4$

**D** 2 only

**E** None of them

## Question 13

Consider the false claim $\frac{d(x^2)}{dx} = x$.

Below is an attempt at the proof.

1. We can write $x^2$ as $x + x + x + ...$ ($x$ times) for any $x$ 2. Given (1), we have $\frac{d(x^2)}{dx} = \frac{d}{dx}(x + x + x + x + ...)$ ($x$ times) 3. Further, $\frac{d}{dx}(x + x + x + ...)$ ($x$ times) is $\frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx}$ ... ($x$ times) 4. And $\frac{d(x)}{dx} + \frac{d(x)}{dx} + \frac{d(x)}{dx}$ ... ($x$ times) simplifies to $1 + 1 + 1 + 1$ ... ($x$ times) 5. ... which is equal to $x$

Which of the above steps is/are wrong?

**A** All of them

**B** $1, 3, 4$

**C** $1, 3$

**D** 1 only

**E** 3 only

## Question 14

A quadrilateral has side lengths 1, 5, 7, and $x$. Which of the following values of $x$ is not possible?

**A** 12

**B** 7

**C** 2

**D** 1

**E** 0.5

## Question 15

Which of the following statements is true?

**A** A positive whole number is divisible by 4 if and only if its square is divisible by 4.

**B** A positive whole number is divisible by 4 if and only if its last two digits form a number divisible by 4.

**C** A positive whole number is divisible by 4 if and only if it has an even number of even factors.

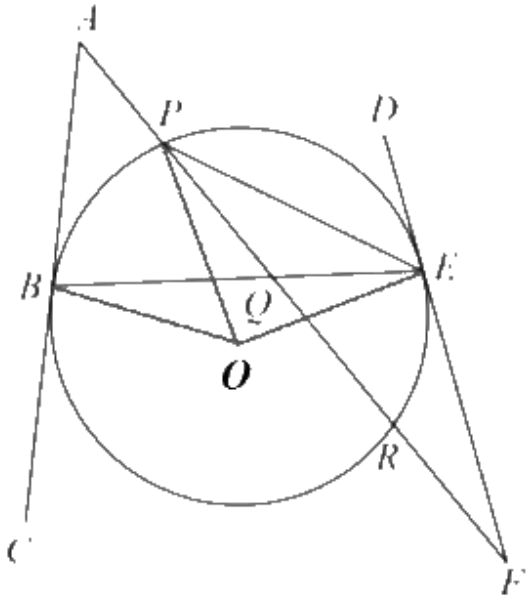
**D** A positive whole number leaves a remainder of 1 when divided by 4 if and only if it is odd.

**E** None of the above

## Question 16

In the diagram, line segments $AC$ and $DF$ are tangent to the circle at $B$ and $E$, respectively. $O$ is the centre of the circle, and radii are drawn to $B$, $P$, and $E$. Also, $AF$ intersects the circle at $P$ and $R$, and intersects $BE$ at $Q$, as shown.

If $\angle CAF = 35°$, $\angle DFA = 30°$, and $\angle FPE = 25°$, determine the angles $PEO$, $BOP$, and $BEP$ in degrees.




**A** $60, 90, 45$

**B** $35, 65, 65$

**C** $35, 65, 32.5$

**D** $45, 75, 75$

**E** $45, 75, 37.5$

## Question 17

For which of the following values of $k$ do the lines $y = 1 - x^2$ and $y = k - 2|x|$ intersect more than twice?

1. $k = 2.5$ 2. $k = 2$ 3. $k = 1.5$ 4. $k = 1$ 5. $k = 0.5$

**A** $2, 3, 4$

**B** $1, 2$

**C** 1 only

**D** $3, 4$

**E** 3 only

## Question 18

Which of the following does not provide a counter example to the claim that $n^3 + n^2 + 111$ is always prime?

**A** $n = 11$

**B** $n = 3$

**C** $n = 1$

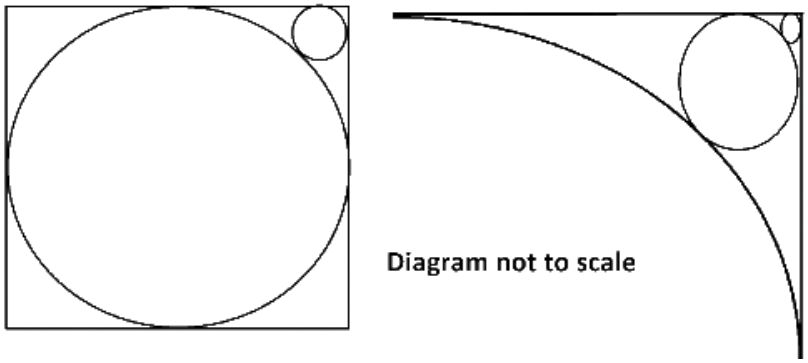
**D** $n = 0$

**E** $n = -3$

## Question 19

A square of side length 2 units contains a circle of radius $R_1$ that is internally tangential to the square. A second smaller circle of radius $R_2$ is inscribed in a corner of the square and is tangential to the circle and the square. Another circle of radius $R_3$ is inscribed in the same corner and is tangential to the smaller circle (radius $R_2$) and the square, and so on ad infinitum for radii $R_4$, $R_5$, etc.

Find $\sum_{t=1}^{\infty} R_t$.




**A** $\frac{\sqrt{2}+1}{2}$

**B** $\frac{\sqrt{2}-1}{2}$

**C** $\sqrt{2}+1$

**D** $3+2\sqrt{2}$

**E** $\frac{3+2\sqrt{2}}{2}$

## Question 20

$p, q, r$ are positive whole numbers and $pq + pr = 80$, while $pq + qr = 425$. Given $p, q, r$ are prime, find the value of $p + q + r$.

**A** 21

**B** 42

**C** 90

**D** 21 or 42

**E** 42 or 90