TMUA
COMMUNITY PAPERS

TMUAmockz Set 1 Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

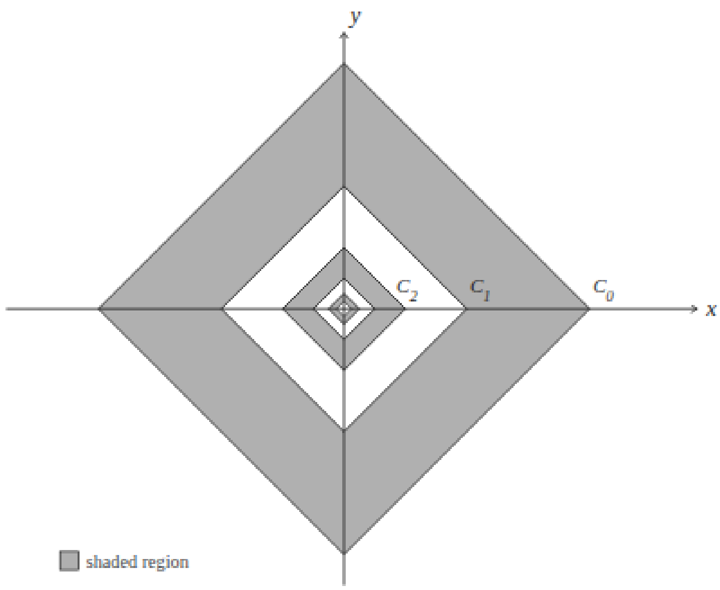
Best of luck, and enjoy the challenge!

Source credit: TMUAmockz

Question 1

The curves C_n are defined by $|x|+|y| = 2^{2-n}$ for non-negative integers n. The region between C_n and C_{n+1} is shaded when n is even, and unshaded when n is odd, as shown in the diagram. This pattern continues infinitely.

Find the total shaded area.



A 8

B 24

C $\frac{32}{3}$

D $\frac{128}{5}$

E $\frac{32}{5}$

F $\frac{64}{5}$

Question 2

Let $f(x) = x^3 - 3px - p(1 + p)$, where $p > 0$ and $p \neq 1$.

How many times does the curve $y = f(x)$ cross the x-axis?

A Always 1

B Always 2

C Always 3

D Either 1 or 2

E Either 1 or 3

F Either 2 or 3

Question 3

Find the area of the region of the coordinate plane satisfying

$$0 \leq x \leq \frac{\pi}{2}, \quad 0 \leq y \leq \frac{\pi}{2}, \quad \text{and} \quad \sin(x+y) \leq \frac{1}{2}.$$

A $\frac{\pi^2}{72}$

B $\frac{\pi^2}{36}$

C $\frac{\pi^2}{18}$

D $\frac{\pi^2}{12}$

E $\frac{\pi^2}{9}$

F $\frac{\pi^2}{6}$

Question 4

How many distinct real solutions are there to the equation $(x^2 + 2x - 3)^{x^2-4} = (x^2 + 2x - 3)^{2x}$?

A 2

B 3

C 4

D 5

E 6

F 7

Question 5

Evaluate

$$\int_{-2}^{4} |x-1|(x^2-2x-3)\,\mathrm{d}x$$

A $-\frac{9}{2}$

B 4

C $\frac{9}{2}$

D 5

E $\frac{41}{2}$

F 0

Question 6

For how many values of the constant k does the equation $(kx^2 - 7x + 6)(x - k) = 0$ have exactly two distinct real roots?

A 2

B 3

C 4

D 5

E 6

F More than 6

Question 7

For some integer $n > 1$, the infinite series $\frac{2}{n} + \frac{3}{n^2} + \frac{2}{n^3} + \frac{3}{n^4} + \cdots = \frac{7}{51}$. Find the value of n.

A 13

B 14

C 15

D 16

E 17

F 18

Question 8

The coefficient of x^2 in the expansion of

$$(1 + 2x + 2x^2)^3$$

is equal to one third of the coefficient of x^2 in the expansion of

$$(1 + mx)^4.$$

Find all possible values of the constant m.

A $m = 3$ only

B $m = \pm 3$

C $m = \pm\sqrt{6}$

D $m = \pm\sqrt{3}$

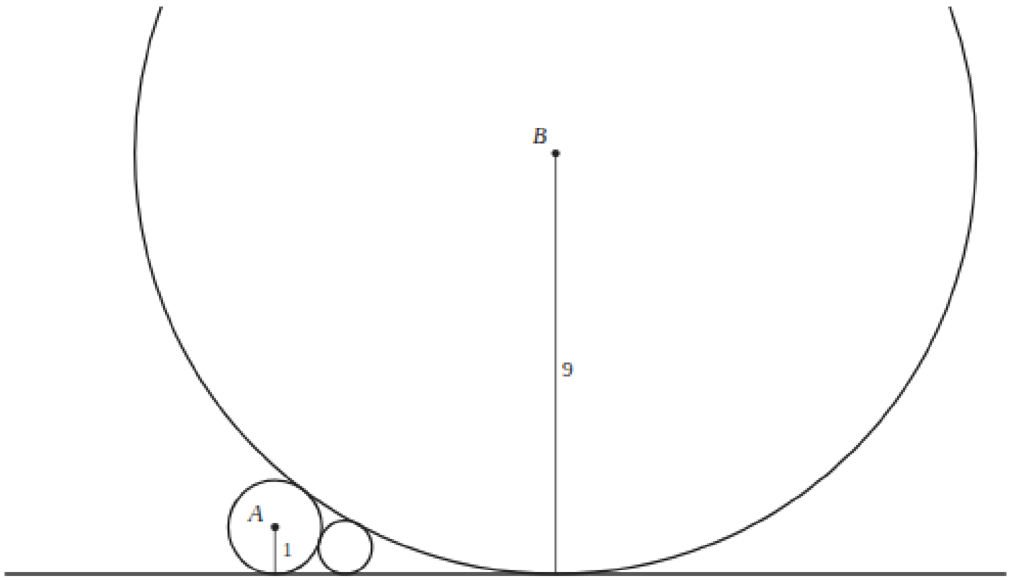
E $m = \pm 2$

F $m = \pm\sqrt{2}$

Question 9

A circle centred at A with radius 1 and a circle centred at B with radius 9 are externally tangent to one another. A third circle is tangent to both circles and to one of their common external tangents, as shown.

Find the radius of the third circle.



A $\frac{1}{2}$

B $\frac{3}{4}$

C $\frac{4}{9}$

D $\frac{9}{16}$

E $\frac{5}{8}$

F $\frac{3}{5}$

Question 10

Two lines $y = x$ and $y = -x$ cross the vertical line $x = 4$. A circle sits in the region to the right of $x = 4$, touching all three lines. Its centre lies on the x-axis. What is its radius?

A $4\sqrt{2} - 4$

B 4

C $4\sqrt{2}$

D 8

E $4 + 4\sqrt{2}$

F $8 + 4\sqrt{2}$

Question 11

A triangle has vertices $A = (1, 2)$, $B = (3, 3)$ and $C = (3m + 1, 2)$, where $m \neq 0$. The line $y - 2 = m(x - 1)$ divides the triangle into two regions of equal area.

Find the sum of all possible values of m.

A $-\frac{1}{3}$

B $\frac{1}{3}$

C $-\frac{1}{2}$

D $-\frac{2}{3}$

E $\frac{2}{3}$

F $-\frac{4}{3}$

Question 12

Find the sum of all values of θ in the interval $0^\circ < \theta < 180^\circ$ that satisfy

$$\cos\theta = \sin 5\theta.$$

A 37.5°

B 225°

C 270°

D 360°

E 315°

F 247.5°

Question 13

An arithmetic progression has first term 25 and common difference d. The numbers obtained by adding 10 to the second term and adding 29 to the third term, whilst leaving the first term unchanged, form a geometric progression.

Find the greatest possible value of the third term of this geometric progression.

A 4

B 25

C 29

D 64

E 79

F 100

Question 14

Triangle PQR has fixed vertices $Q = (0, 3)$ and $R = (4, 0)$. The third vertex $P = (a, b)$ has positive integer coordinates satisfying $a + 2b = 14$.

What is the greatest possible area of triangle PQR?

A 10

B 11

C 12

D 13

E 14

F 15

Question 15

Evaluate $\sum_{k=1}^{49} \sin^2\left(\frac{k\pi}{50}\right)$.

A 0

B $\frac{1}{2}$

C $\frac{49}{2}$

D 25

E 49

F 50

Question 16

The function $g(x) = \lfloor x \rfloor \cdot \lceil x \rceil$ is defined for all real x, where $\lfloor x \rfloor$ denotes the floor of x and $\lceil x \rceil$ the ceiling of x.

Find the value of $\displaystyle\int_0^{10} g(x)\,\mathrm{d}x$.

A 285

B 300

C 330

D 385

E 550

F 1000

Question 17

A regular hexagon $PQRSTU$ has vertex P at $(1, 1)$ and vertex R at $(7, 5)$.

Find the area of the hexagon.

A $20\sqrt{3}$

B $22\sqrt{3}$

C $24\sqrt{3}$

D $26\sqrt{3}$

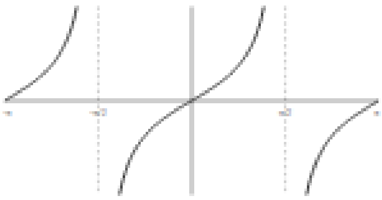
E $28\sqrt{3}$

F 52

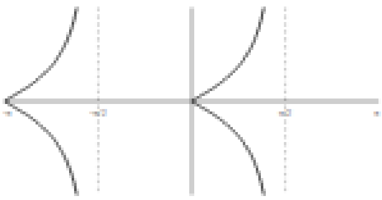
Question 18

Which of the following graphs best represents the curve with equation $|y| = -\tan x$ in the range $-\pi < x < \pi$ (excluding points where $\tan x$ is undefined)?

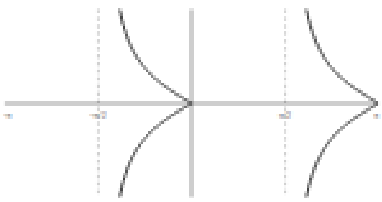
A A



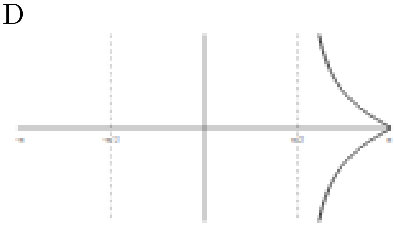
B B



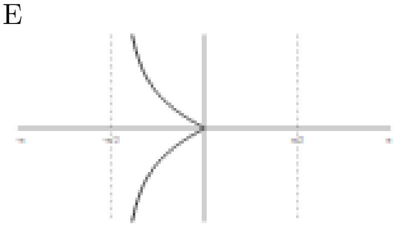
C C



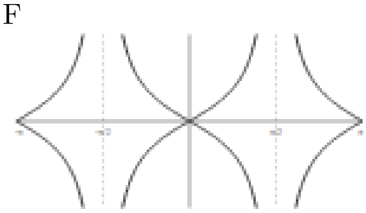
D D



E E



F F



Question 19

Positive integers p, q and r satisfy $p < q < r$. The simultaneous equations

$$2x + y = 12 \quad \text{and} \quad y = |x - p| + |x - q| + |x - r|$$

have exactly one solution. Find the minimum possible value of r.

A 4

B 5

C 6

D 7

E 8

F 9

Question 20

The equation

$$16x^4 - 12x^2 + 1 = 0$$

has four real solutions. The second largest solution can be written in the form

$$\frac{p + q\sqrt{5}}{4}$$

where p and q are integers. Find the value of $p + q$.

A -2

B -1

C 0

D 2

E 3

F 4