TMUA

COMMUNITY PAPERS

Thriving Scholars TMUA Full Mock Test 1 Paper 1

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Thriving Scholars

Question 1

A square has centre $(3, 4)$ and one corner at $(1, 5)$. Another corner is at

A $(1, 3)$

B $(5, 5)$

C $(2, 2)$

D $(5, 2)$

E $(4, 2)$

Question 2

What is the value of $\int_0^1 (e^x - x)(e^x + x)\,\mathrm{d}x$?

A $\frac{3e^2-5}{6}$

B $\frac{e^2+3}{6}$

C $\frac{3e^2-2}{6}$

D $\frac{2e^2-3}{6}$

E $\frac{3e^2+2}{6}$

Question 3

A regular dodecagon is a 12-sided polygon with all sides the same length and all internal angles equal. If I construct a regular dodecagon by connecting 12 equally-spaced points on a circle of radius 1, then the area of this polygon is

A 3

B $3\sqrt{3}$

C $3\sqrt{2}$

D $2\sqrt{2}$

E $6+3\sqrt{3}$

Question 4

The positive number a satisfies

$$\int_0^a \left(\sqrt{x} + x^2\right)\, \mathrm{d}x = 5$$

if

A $a = \sqrt{3}$

B $a = \left(\sqrt{21} - 1\right)^{1/3}$

C $a = 3^{2/3}$

D $a = 5^{2/3}$

E $a = \left(\sqrt{6} - 1\right)^{2/3}$

Question 5

Tangents to $y = e^x$ are drawn at (p, e^p) and (q, e^q). These tangents cross the x-axis at a and b, respectively. It follows that, for all p and q

A $pa = qb$

B $p - a = q - b$

C $p - a < q - b$

D $p + q = a + b$

E $p - a > q - b$

Question 6

How many real solutions x are there to the equation $x|x| + 1 = 3|x|$?

[Note that $|x|$ is equal to x if $x \geq 0$, and equal to $-x$ otherwise.]

A 1

B 3

C 4

D 0

E 2

Question 7

If the expression $\left(x+1+\frac{1}{x}\right)^4$ is fully expanded term-by-term and like terms are collected together, there is one term which is independent of x. The value of this term is

A 51

B 81

C 10

D 19

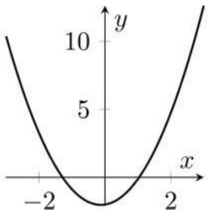
E 14

Question 8

The following five graphs are, in some order, plots of $y = f(x)$, $y = g(x)$, $y = h(x)$, $y = \dfrac{\mathrm{d}f}{\mathrm{d}x}$ and $y = \dfrac{\mathrm{d}g}{\mathrm{d}x}$; that is, three unknown functions and the derivatives of the first two of those functions. Which graph is a plot of $h(x)$?

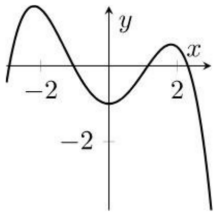
A A

A



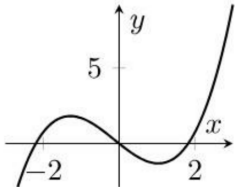
B B

B



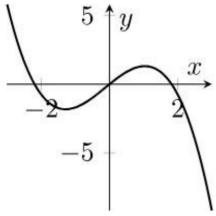
C C

C



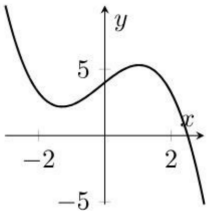
D D

D



E E

E



Question 9

Let s_k denote the sum of the kth powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. In particular, $s_0 = 3$, $s_1 = 5$, and $s_2 = 9$. Let a, b, and c be real numbers such that $s_{k+1} = a\,s_k + b\,s_{k-1} + c\,s_{k-2}$ for $k = 2, 3, ...$ What is $a + b + c$?

A -6

B 0

C 6

D 10

E 26

Question 10

In $\triangle ABC$ with integer side lengths, $\cos A = \frac{11}{16}$, $\cos B = \frac{7}{8}$, and $\cos C = -\frac{1}{4}$. What is the least possible perimeter for $\triangle ABC$?

A 9

B 12

C 23

D 27

E 44

Question 11

Define binary operations $\diamond$ and $\heartsuit$ by

$$a \diamond b = a^{\log_7(b)} \qquad \text{and} \qquad a \heartsuit b = a^{\frac{1}{\log_7(b)}}$$

for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and

$$a_n = (n \heartsuit (n-1)) \diamond a_{n-1}$$

for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?

A 8

B 9

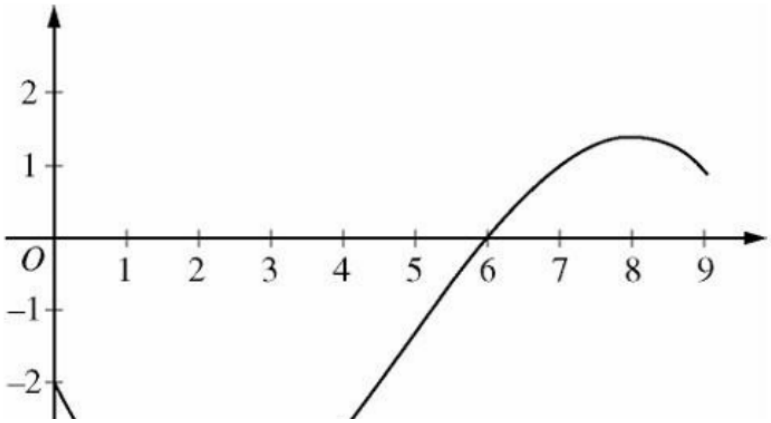
C 10

D 11

E 12

Question 12

The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?



A $h(6) < h'(6) < h''(6)$

B $h(6) < h''(6) < h'(6)$

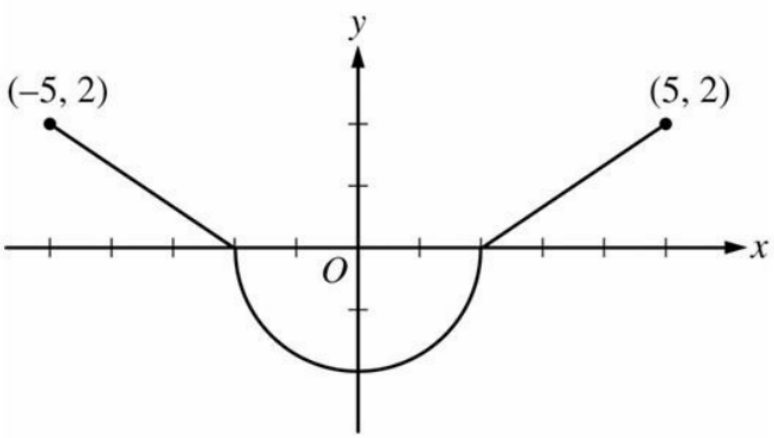
C $h'(6) < h(6) < h''(6)$

D $h''(6) < h(6) < h'(6)$

E $h''(6) < h'(6) < h(6)$

Question 13

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$



Graph of f'

A $2\pi - 2$

B $2\pi - 3$

C $2\pi - 5$

D $6 - 2\pi$

E $4 - 2\pi$

Question 14

The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

A $(0, 0)$ only

B $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

C $(0, 0)$ and $(-4, 2)$

D $(0, 0)$ and $\left(4, \frac{2}{3}\right)$

E There are no such points.

Question 15

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

A 2020

B 4039

C 6057

D 6061

E 8078

Question 16

Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,\mathrm{d}t$. Which of the following must be true on the interval $0 < x < 2$?

A g is increasing, and the graph of g is concave up.

B g is increasing, and the graph of g is concave down.

C g is decreasing, and the graph of g is concave up.

D g is decreasing, and the graph of g is concave down.

E g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

Question 17

Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2(x/y))^2$?

A $\frac{25}{2}$

B 20

C $\frac{45}{2}$

D 25

E 32

Question 18

How many solutions does $\tan(2x) = \cos(\frac{x}{2})$ have on the interval $[0, 2\pi]$?

A 1

B 2

C 3

D 4

E 5

Question 19

What is the sum of all possible values of k for which the polynomials $x^2 - 3x + 2$ and $x^2 - 5x + k$ have a root in common?

A 3

B 4

C 5

D 6

E 10

Question 20

An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius 1. An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π?

A 46

B 48

C 56

D 60

E 64