



Thriving Scholars TMUA Full Mock Test

1 Paper 2

Recommended time: 75 minutes

Calculators: not permitted

Format: 20 multiple-choice questions

This community paper is presented in a clean, print-ready format. The wording, mathematics, answer choices, and diagrams are preserved from the source paper.

Paper guidance

- Attempt every question. Select the best answer from the choices given.
- If a diagram is shown, it is reproduced from the source material and may not be drawn to scale.
- This question paper does not include worked solutions or a marked answer key.

Best of luck, and enjoy the challenge!

Source credit: Thriving Scholars

Question 1

A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11?

A $\frac{4}{100}$

B $\frac{9}{200}$

C $\frac{1}{20}$

D $\frac{11}{200}$

E $\frac{3}{50}$

Question 2

Let $P(x)$ be the unique polynomial of minimal degree with the following properties:

- $P(x)$ has a leading coefficient 1,
- 1 is a root of $P(x) - 1$,
- 2 is a root of $P(x - 2)$,
- 3 is a root of $P(3x)$, and
- 4 is a root of $4P(x)$.

The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written as $\frac{m}{n}$, where m and n are relatively prime integers. What is $m + n$?

- A** 41
- B** 43
- C** 45
- D** 47
- E** 49

Question 3

How many three-digit positive integers N satisfy the following properties?

- The number N is divisible by 7.
- The number formed by reversing the digits of N is divisible by 5.

A 13

B 14

C 15

D 16

E 17

Question 4

Let $f(x) = 3 - 2x$. A differentiable function g satisfies $g'(x) = f(x)f'(x)(x - 3)$.

For this question, take "g is increasing at x " to mean $g'(x) \geq 0$.

Decide which of the following statements are true.

1. $x \leq \frac{3}{2}$ is a **sufficient** condition for g to be increasing at x . 2. $x \geq 3$ is a **necessary** condition for g to be increasing at x . 3. $x \leq \frac{3}{2}$ or $x \geq 3$ is a **necessary and sufficient** condition for g to be increasing at x .

How many of the statements 1-3 are true?

A 0

B 1

C 2

D 3

Question 5

Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?

- A 4
- B 5
- C 6
- D 7
- E 8

Question 6

What is the least value of n such that $n!$ is a multiple of 2024?

- A 11
- B 21
- C 22
- D 23
- E 253

Question 7

How many angles θ with $0 \leq \theta \leq 2\pi$ satisfy $\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0$?

A 0

B 1

C 2

D 3

E 4

Question 8

On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

"Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

"Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.

"Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes.

How many pieces of candy in all did the principal give to the children who always tell the truth?

- A 7
- B 12
- C 21
- D 27
- E 31

Question 9

Which expression is equal to

$$|a - 2 - \sqrt{(a - 1)^2}|$$

for $a < 0$?

A $3 - 2a$

B $1 - a$

C 1

D $a + 1$

E 3

Question 10

What is the least possible value of $(xy - 1)^2 + (x + y)^2$ for real numbers x and y ?

A 0

B $\frac{1}{4}$

C $\frac{1}{2}$

D 1

E 2

Question 11

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \dots, 200, 200, \dots, 200$

What is the median of the numbers in this list?

- A** 100.5
- B** 134
- C** 142
- D** 150.5
- E** 167

Question 12

What is the sum of all real numbers x for which $|x^2 - 12x + 34| = 2$?

- A 12
- B 15
- C 18
- D 21
- E 25

Question 13

What is the value of

$$1 + 2 + 3 - 4 + 5 + 6 + 7 - 8 + \cdots + 197 + 198 + 199 - 200?$$

- A** 9,800
- B** 9,900
- C** 10,000
- D** 10,100
- E** 10,200

Question 14

Four distinct real numbers a , b , c , and d are used to define four points

$$A = (a, b), \quad B = (b, c), \quad C = (c, d), \quad D = (d, a).$$

The quadrilateral $ABCD$ has all four sides the same length

- A** if and only if $(a - b)^2 = (c - d)^2$
- B** for no values of a , b , c , d
- C** if and only if $(a - d)^2 = (b - c)^2$
- D** if and only if $a - b + c - d = 0$
- E** if and only if $(a - c)^2 = (b - d)^2$

Question 15

Tom runs a small reptile education booth at the weekend market. He keeps notes on 13 snakes: 4 are purple and 5 are happy. From observing their "math tricks," he knows that

- all of his happy snakes can add,
- none of his purple snakes can subtract, and
- any snake that can't subtract also can't add.

Which of these conclusions must be true?

- A** Being purple is a necessary condition for being happy.
- B** Being able to subtract is a sufficient condition for being happy.
- C** Being able to add is a sufficient condition for being happy.
- D** Not being purple is a necessary condition for being happy.
- E** Not being able to subtract is a necessary condition for being happy.

Question 16

Two transformations are said to commute if applying the first followed by the second gives the same result as applying the second followed by the first. Consider these three transformations of the coordinate plane:

- a translation 2 units to the right,
- a 90° -rotation counterclockwise about the origin,
- a reflection across the x -axis.

Of the 3 pairs of distinct transformations from this list, how many commute?

- A** 1
- B** 2
- C** 3
- D** None

Question 17

Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where $\log$ denotes the base 10 logarithm. What is ab ?

A 10^{52}

B 10^{100}

C 10^{144}

D 10^{164}

E 10^{200}

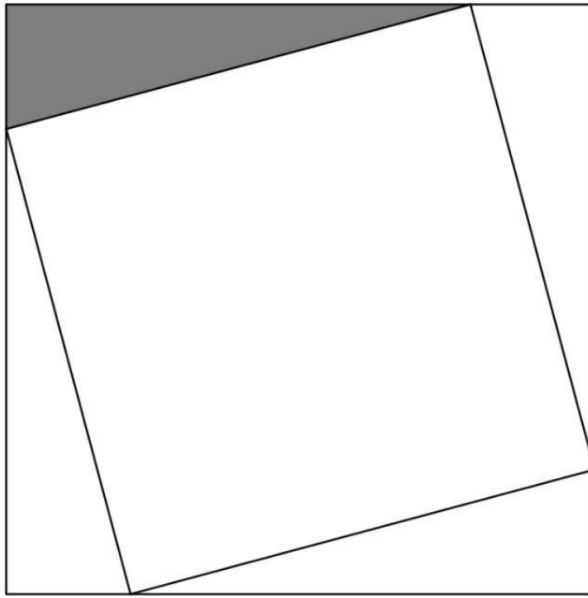
Question 18

The equation $x^2 - 4kx + y^2 - 4y + 8 = k^3 - k$ is the equation of a circle

- A** if and only if either $-4 < k < -1$ or $k > 1$
- B** if and only if $k < -1$
- C** for all real values of k
- D** if and only if $k > 1$
- E** if and only if either $-1 < k < 0$ or $k > 1$

Question 19

A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



- A $\frac{1}{5}$
- B $\frac{1}{4}$
- C $2 - \sqrt{3}$
- D $\sqrt{3} - \sqrt{2}$
- E $\sqrt{2} - 1$

Question 20

Define

$$P(x) = (x - 1^2)(x - 2^2) \cdots (x - 100^2).$$

How many integers n are there such that $P(n) \leq 0$?

- A** 4900
- B** 4950
- C** 5000
- D** 5050
- E** 5100